Transverse Convective Instabilities of a Bunch with Space Charge

Alexey Burov

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Discussions with E. Metral, Yu. Alexahin and T. Zolkin are appreciated.

APT talk, Mar 5, 2019

The Problem

1. Single bunch in a circular machine
2. Linear optics
3. Zero chromaticity
4. Short Wake
5. Space charge

What sorts of modes are there?

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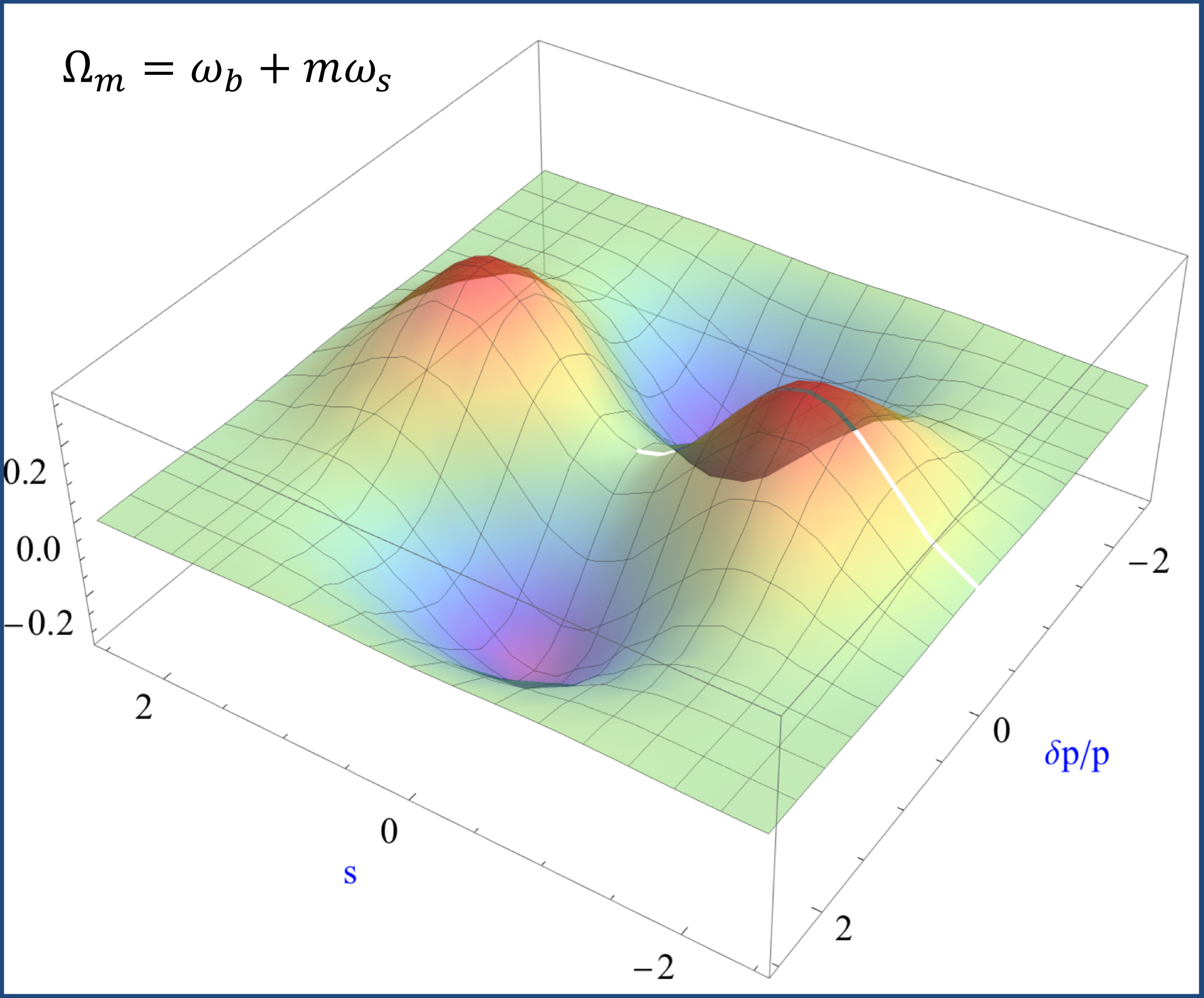
No SC, no Wake



Sacherer's HT mode {2,1}

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Frank Sacherer (1940-1978)

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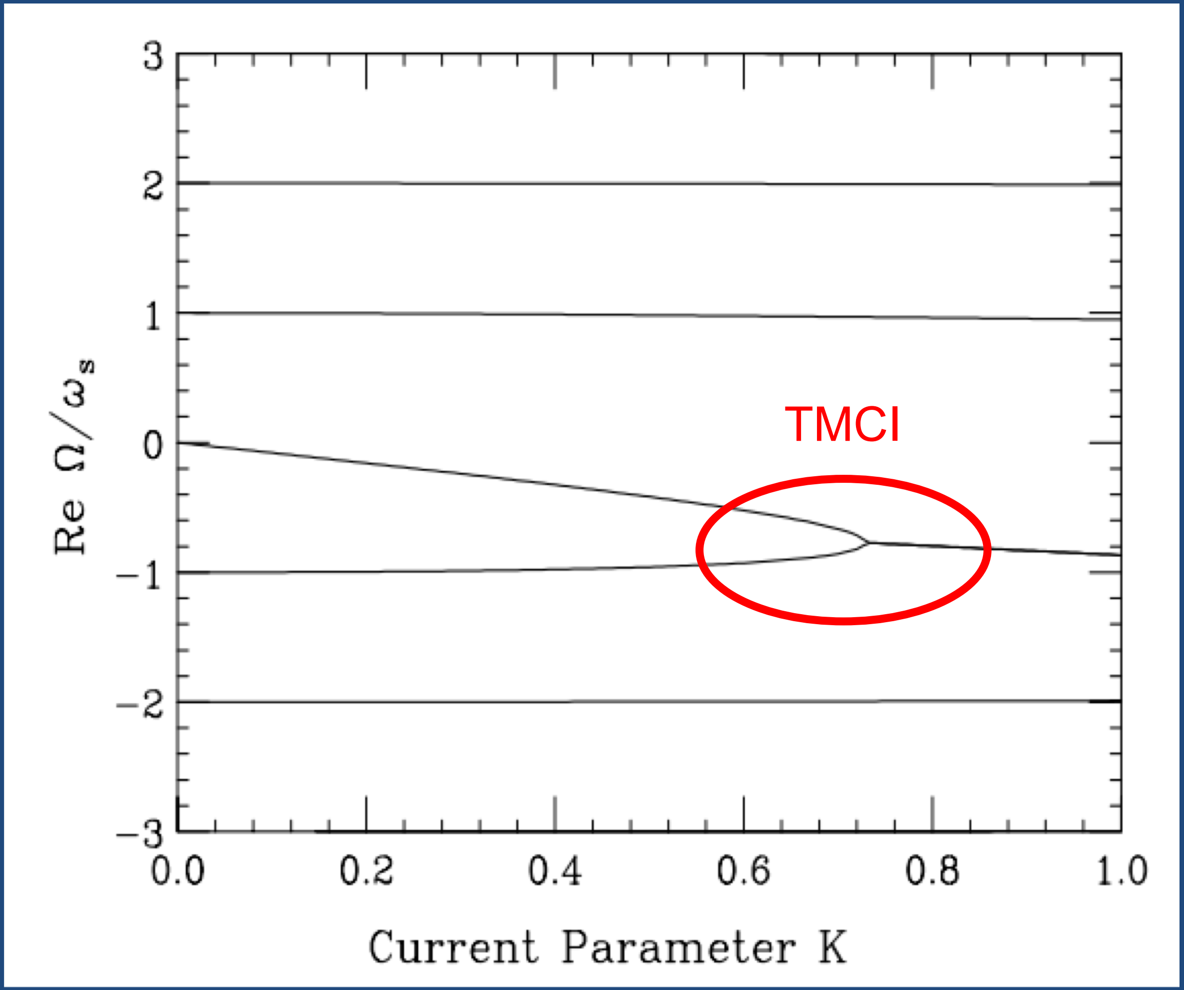
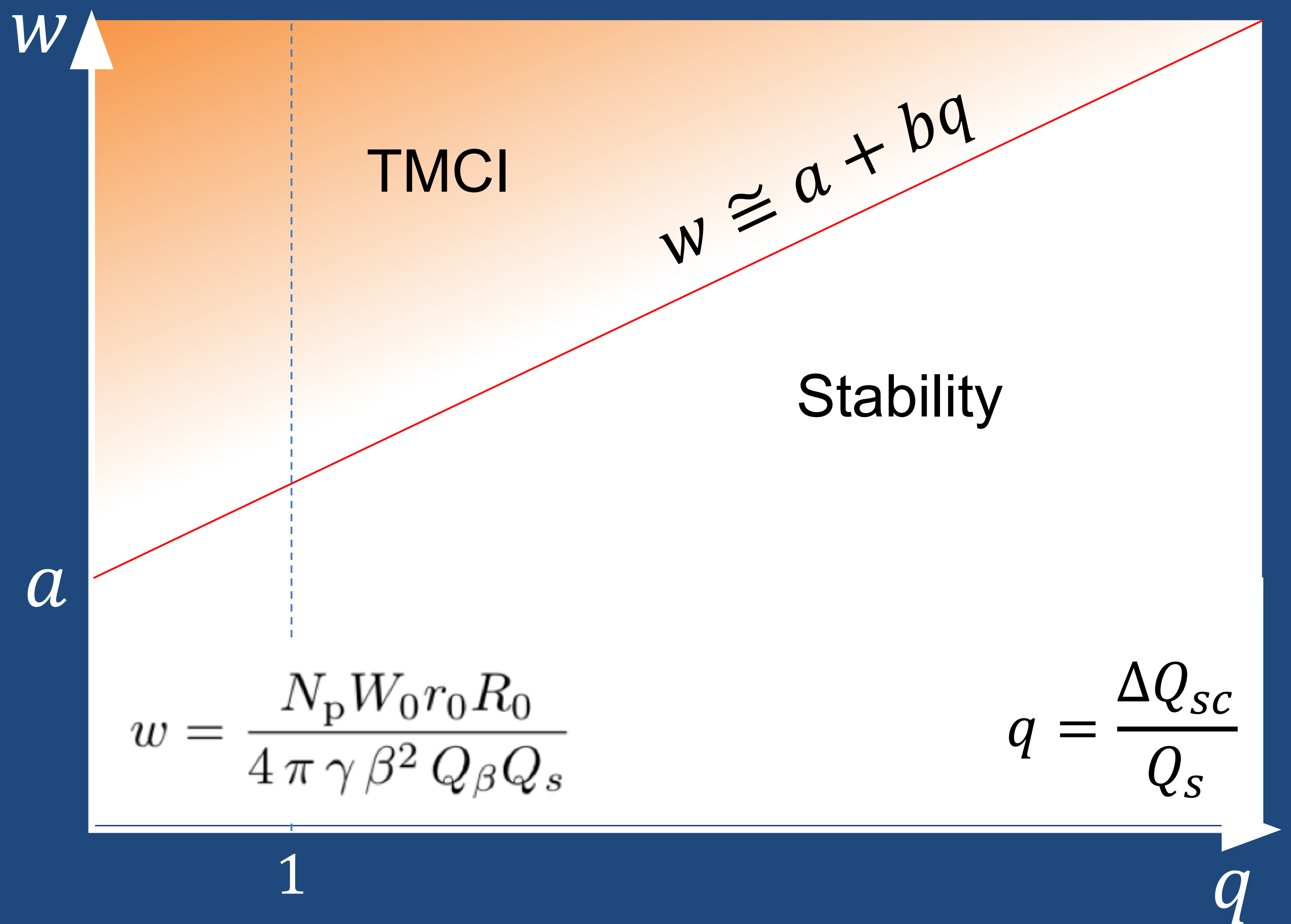
Wake, No SC



First detected at PETRA; explained by R.D. Kohaupt (DESY), 1980

TMCI @ SC



Blaskiewicz (1998), Burov (1999+), Balbekov (2009+), Burov & Zolkin (2017)

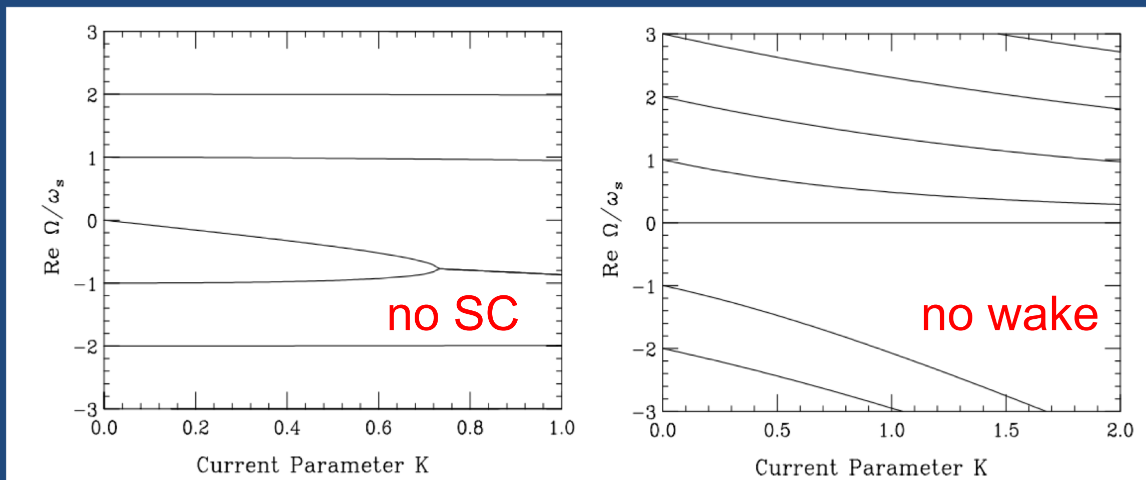
Stability Issues of Low-Energy Intense Beams

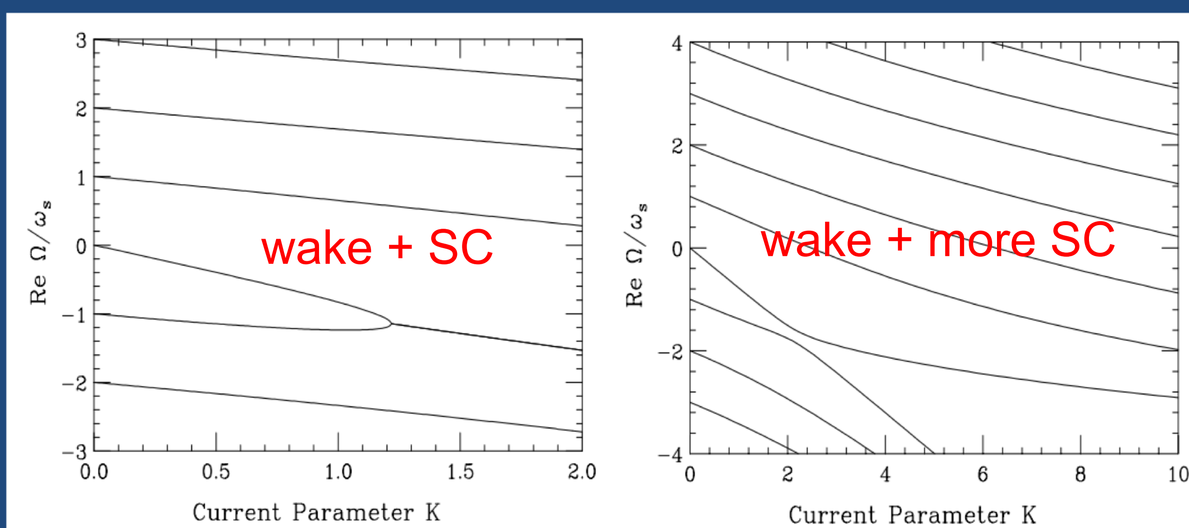
K.Y. Ng and A.V. Burov

FERMILAB-FN-685

October 1999

Explanation of the TMCI suppression





$$N_{th} \propto \frac{1}{w/N - q/N}$$

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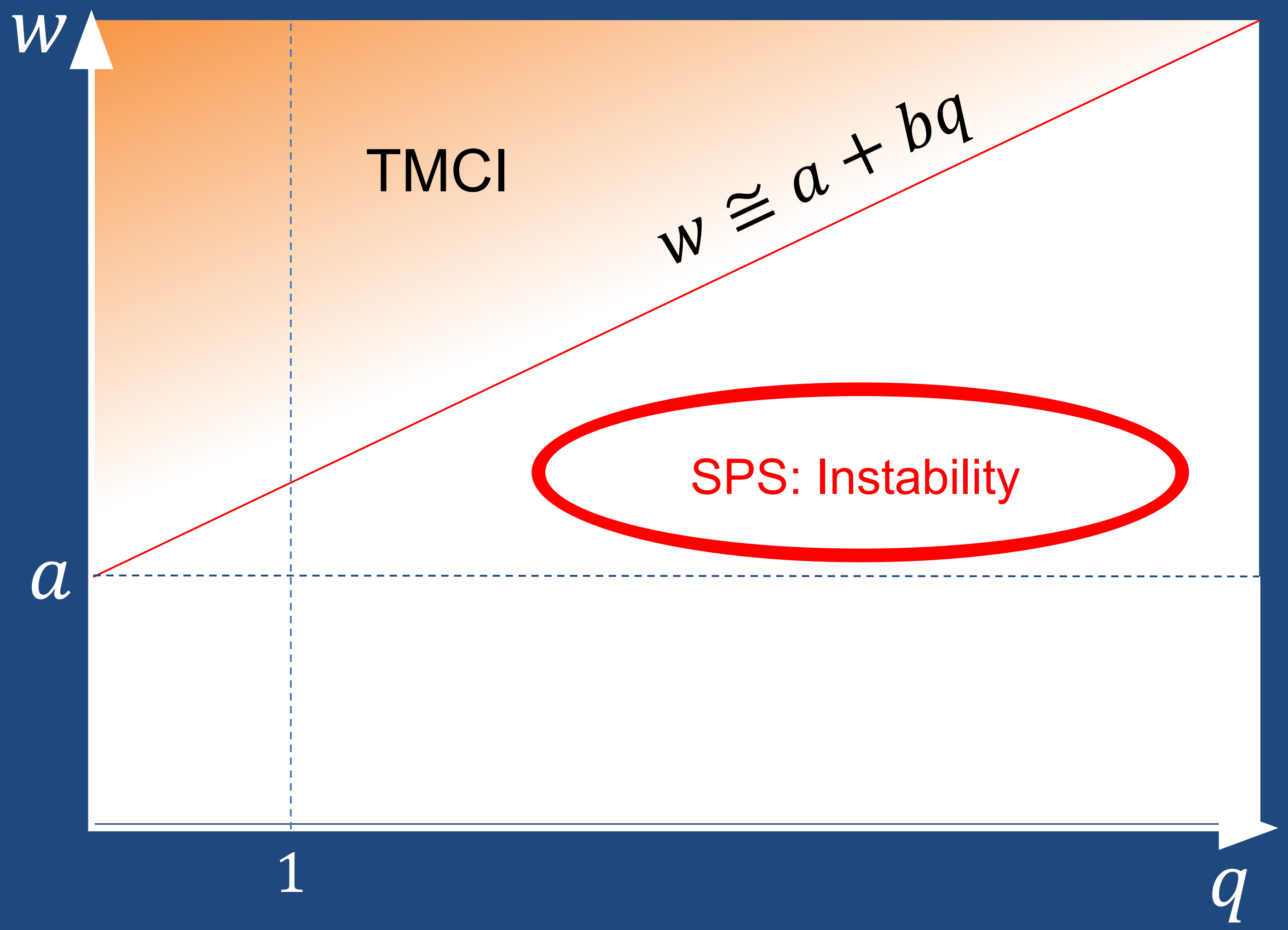
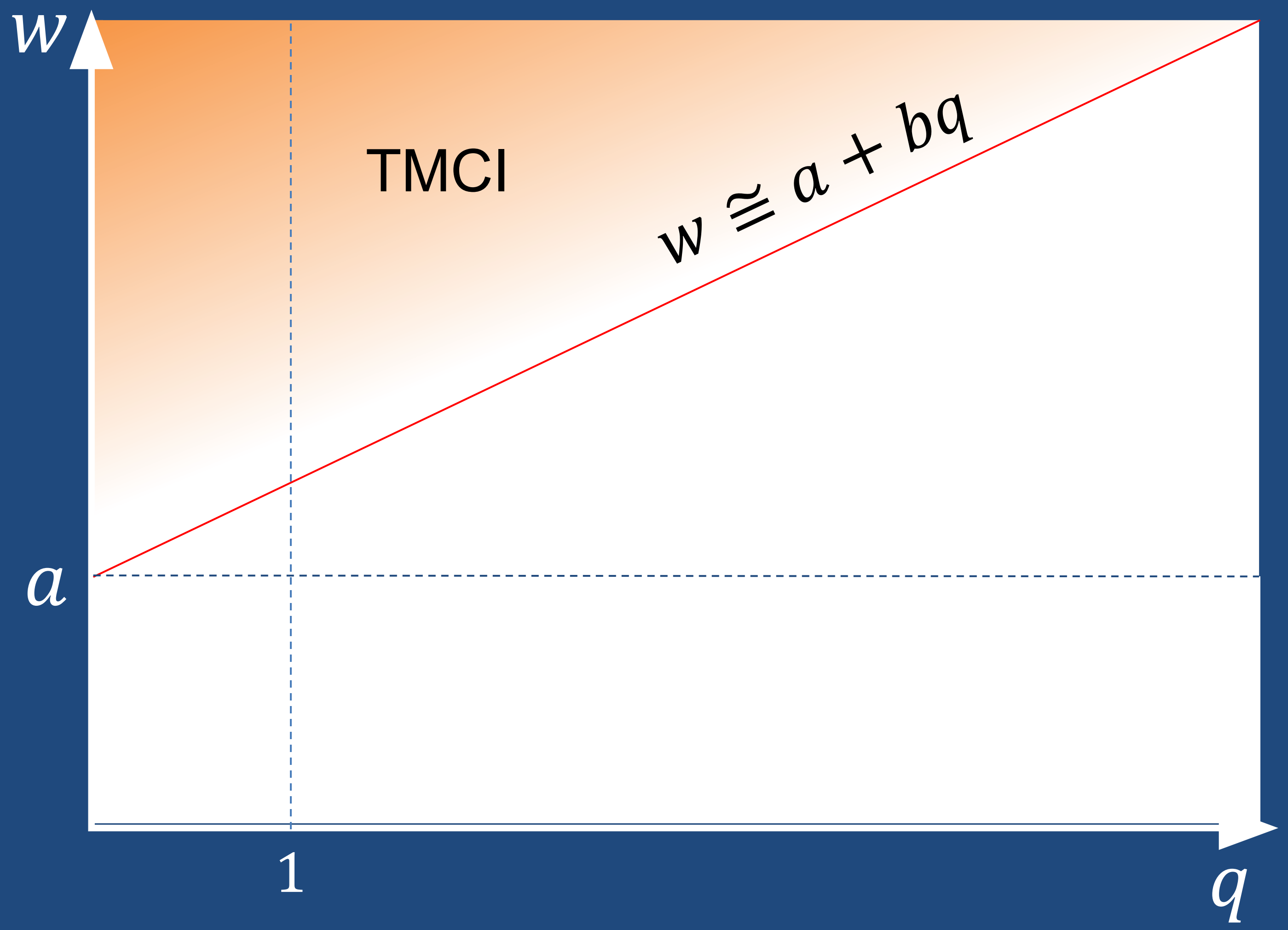
Contradiction #1



Contradiction #2



At SSC, TMCI threshold $w \propto q$ is intensity independent!

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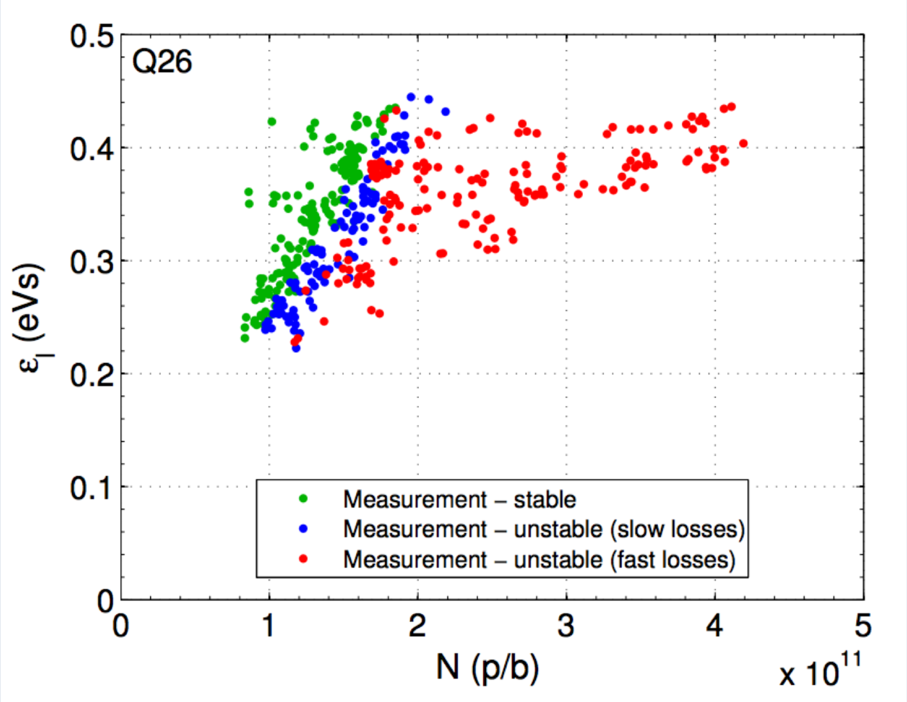
SPS Instability

H. Bartosik, *Beam dynamics and optics studies for the LHC injectors upgrade*, Ph.D. thesis, TU Vienna (2013-10-23).



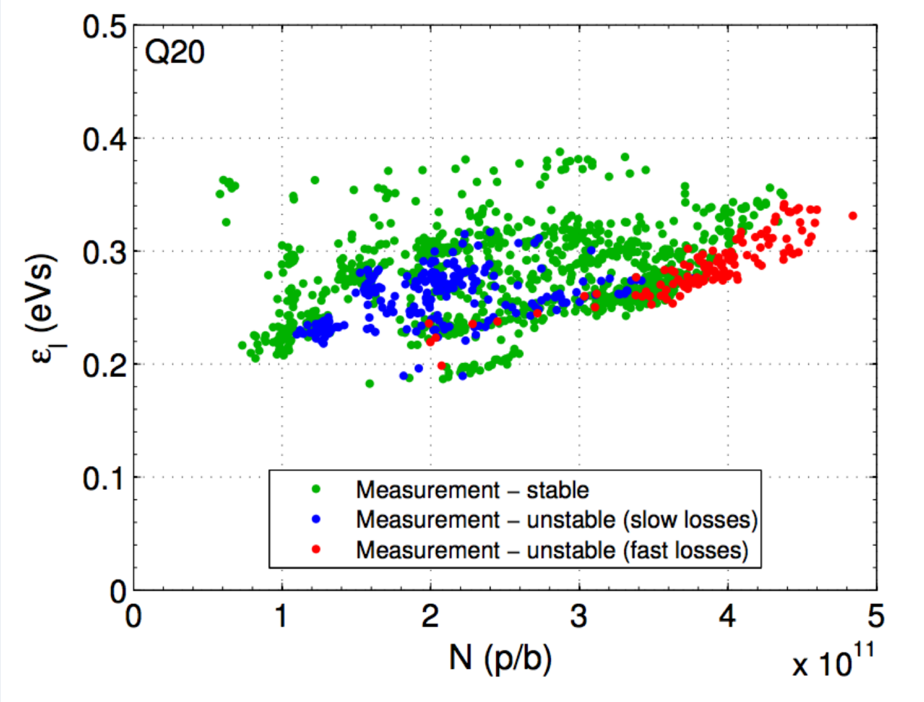


Figure 4.17: Experimental study of beam stability at injection with the Q26 optics (top) and Q20 optics (bottom). Each data point shows the longitudinal emittance measured in the PS before bunch rotation as a function of the beam intensity at the PS extraction, where the color code indicates if the beam was stable in the SPS. Green points correspond to stable cases. Unstable cases are marked by red dots if losses occurred within the first 1000 turns after injection ("fast losses") and blue dots if losses occurred later in the cycle.

These measurements reasonably agree with no SC TMCI threshold, but SC was really huge at Q26, q > 20.
These measurements contradict to everything in the TMCI @ SSC theory!

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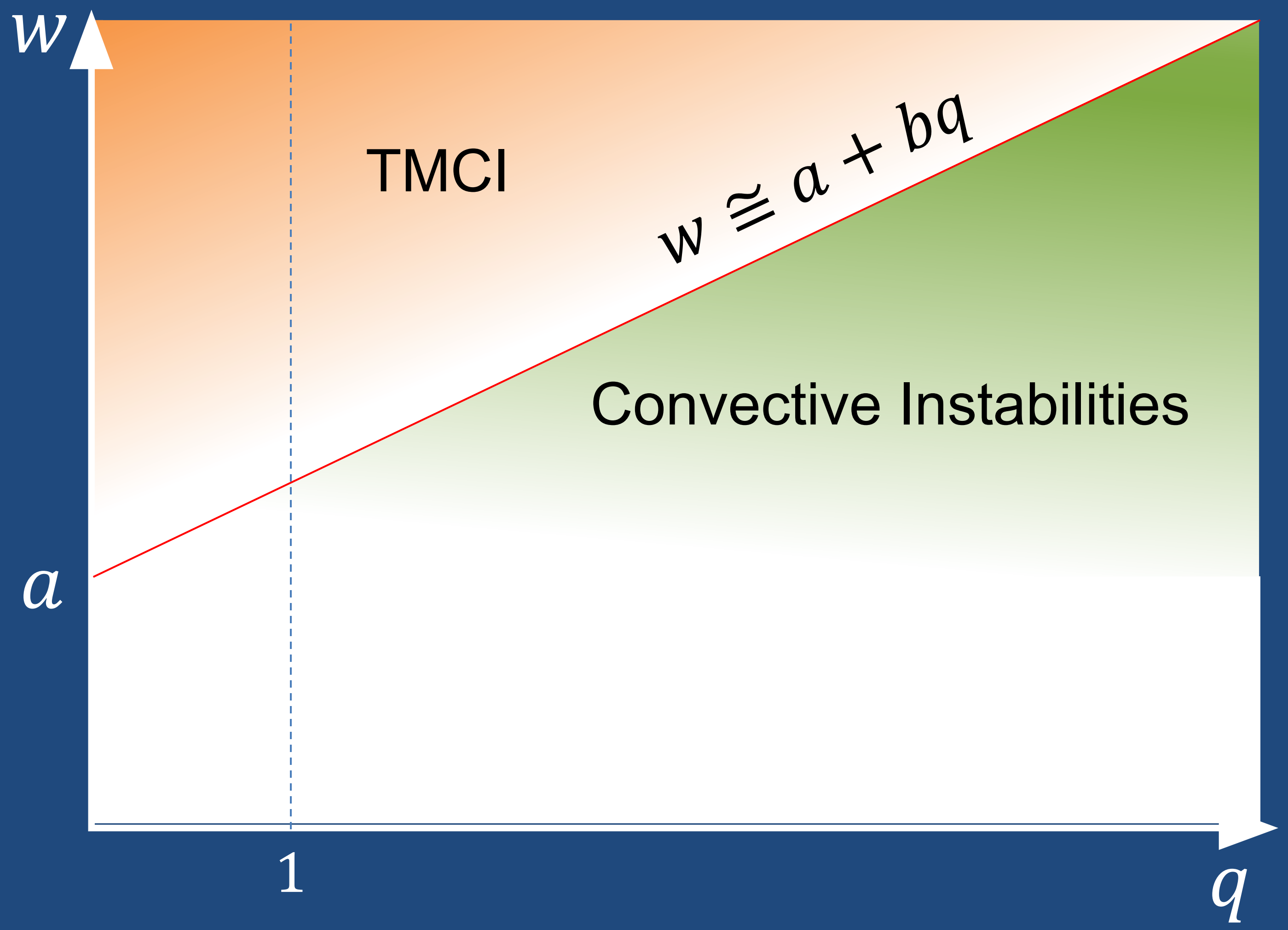
Resolution of the Conundrum



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Resolution

Convective Instabilities of Bunched Beams with Space Charge

A. Burov*
Fermilab, PO Box 500, Batavia, IL 60510-5011
(Dated: August 15, 2018)

Although the transverse mode coupling instability (TMCI) threshold normally increases linearly with the space charge tune shift, it is hard to benefit from that significantly: while the space charge (SC) suppresses TMCI, it introduces *saturating convective* and *absolute-convective* instabilities, SCI and ACI, which could make the beam even less stable than without SC. Due to this a convective instability should develop near the transition energy of hadron machines, while TMCI should be suppressed there. In particular, either SCI or ACI may be an explanation of SC-independence of the transverse instability at CERN SPS under its old Q26 optics, so far unexplained.

arXiv:1807.04887

Beam stability requires more than $\Im\,\nu \leq 0$!

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Air-Bag, Square well (ABS)

$$\frac{\partial x^+}{\partial \theta} - \frac{1}{\pi}\frac{\partial x^+}{\partial s} = \frac{iq}{2}(x^+ - x^-) + i\,F,$$
$$\frac{\partial x^-}{\partial \theta} + \frac{1}{\pi}\frac{\partial x^-}{\partial s} = \frac{iq}{2}(x^- - x^+) + i\,F, \quad (1)$$

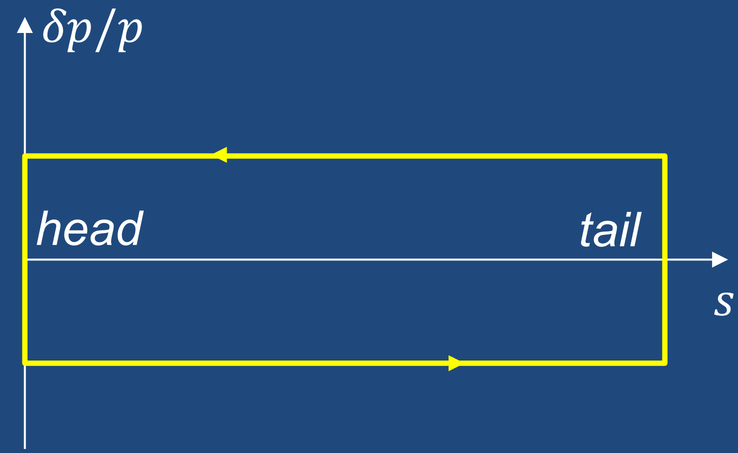
$$F(\theta, s) = w\int_0^s \mathrm{d}s' W(s - s')\frac{x^+(\theta, s') + x^-(\theta, s')}{2}\,, \quad (2)$$

with the boundary conditions

$$x^+ = x^- \quad \text{at } s = 0, 1\,. \quad (3)$$

Here the SC parameter q is the ratio of the SC tune shift to the synchrotron tune, and w is the wake parameter:

$$w = \frac{N_\mathrm{p} W_0 r_0 R_0}{4\,\pi\,\gamma\,\beta^2\,Q_\beta Q_s}. \quad (4)$$



ABS: longitudinal plane

M. Blaskiewicz, 1998

Air-Bag, Square well (ABS)

$$x(\psi) = \begin{cases} x^+(s)\,, & \text{with } \psi = -\pi s\,, \quad -\pi \le \psi \le 0\,; \\ x^-(s)\,, & \text{with } \psi = \pi s\,, \quad 0 \le \psi \le \pi\,. \end{cases}$$

$$\frac{\partial x}{\partial \theta} + \frac{\partial x}{\partial \psi} = \frac{iq}{2}\left[x(\psi) - x(-\psi)\right] + i\,F,$$

$$x(\theta,\psi) = \sum_{n=-\infty}^{\infty} A_n(\theta)\exp(i\,n\psi)$$

$$i\dot{A}_n = n - \frac{q}{2}(A_n - A_{-n}) - w\sum_{m=-\infty}^{\infty} U_{nm}A_m\,,$$

$$U_{nm} \equiv \int_0^1 ds \int_0^s ds' W(s-s')\cos(\pi n s)\cos(\pi m s')$$

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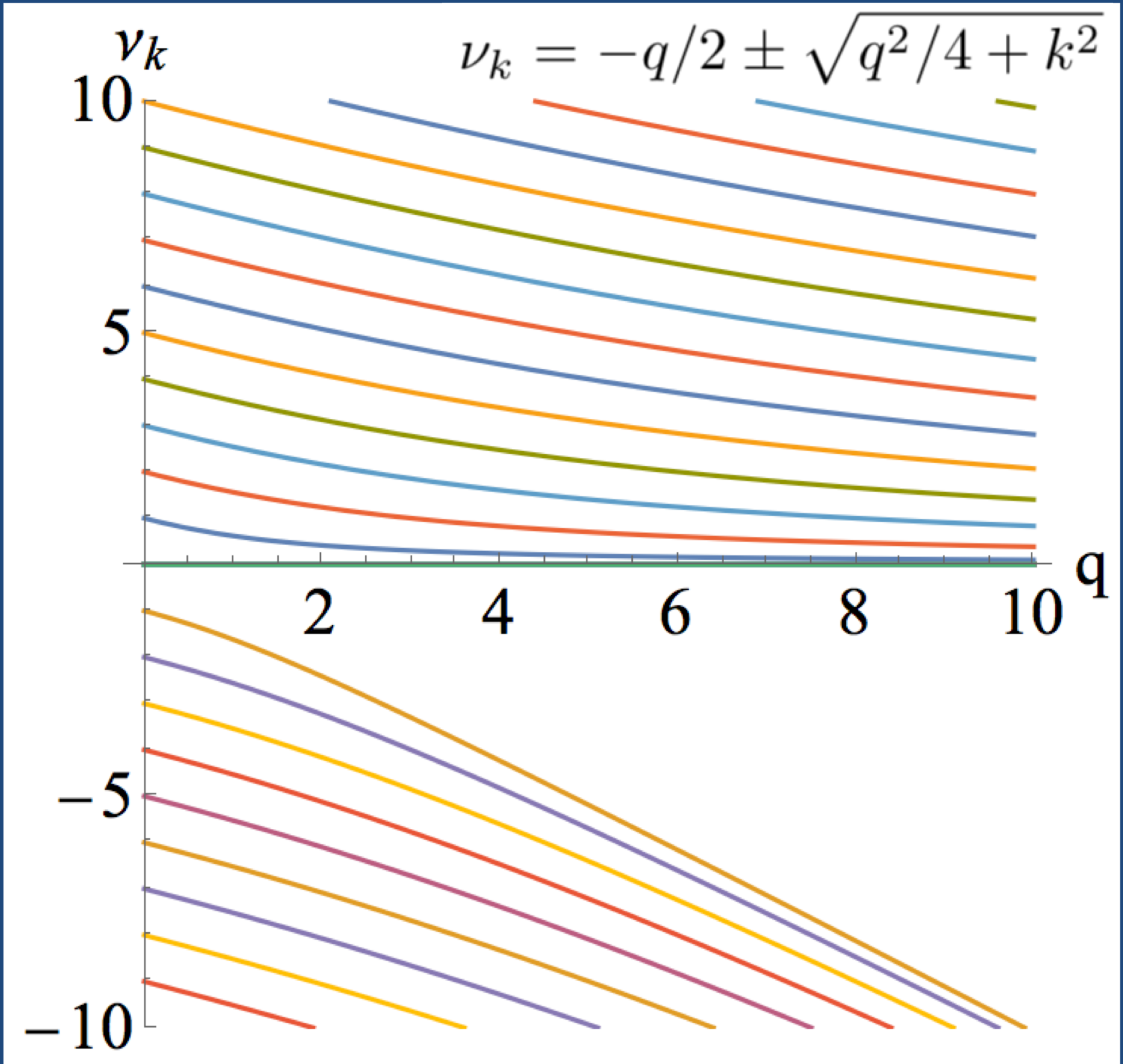
No-Wake Eigenvalues



No-Wake Eigensystem

$$\nu_k = -q/2 \pm \sqrt{q^2/4 + k^2}\,;$$
$$x_k^{\pm}(s) = C_k(\cos(k\pi s) \mp i\,\sin(k\pi s)\,\nu_k/k)$$

$$\int_0^1 ds \frac{|x_k^+|^2 + |x_k^-|^2}{2} = \sum_{n=-\infty}^{\infty} |A_{nk}|^2 = 1$$

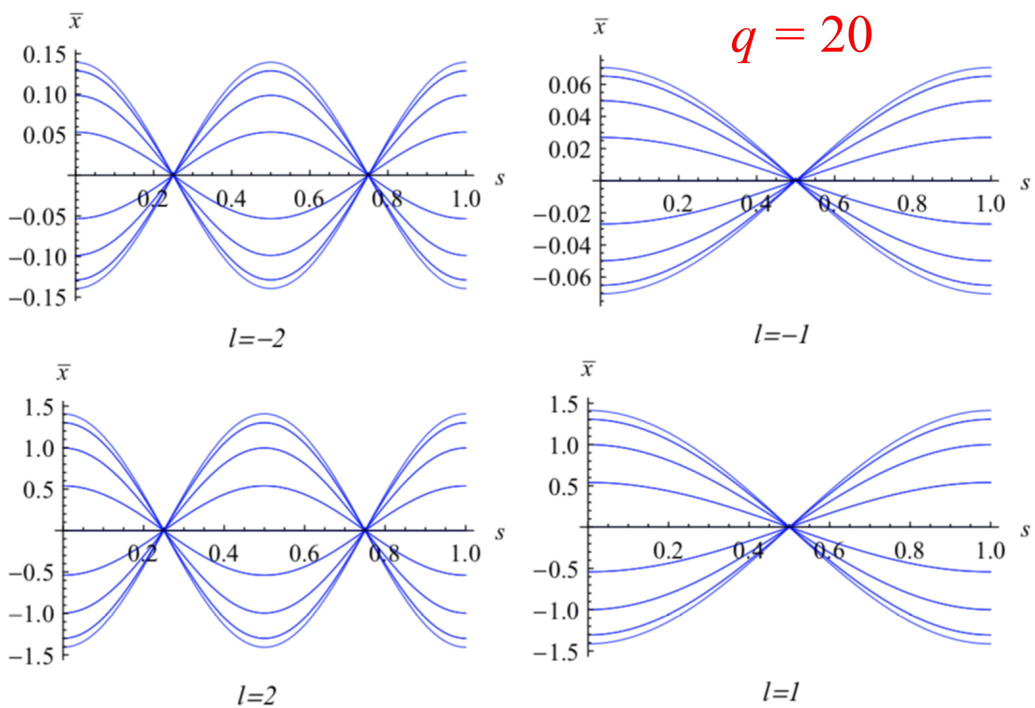


FIG. 2. Stroboscopic snapshots of the centroid oscillations for the same case, $w = 0$, $q = 20$ and modes, $l = \pm 1, \pm 2$, as Fig. 1 above. The opposite modes, l and $-l$, show the same pattern, $\bar{x}_l(s) \propto \cos(\pi l s)$, but the amplitudes differ by a large factor $|l|/q$, reflecting almost in phase oscillations of x^+ and x^- for the positive modes and almost out of phase ones for the negative modes.

$$\Re(\bar{x}(s)\exp(-i\theta_j))$$

$$\theta_j = 2\pi j/N_s,\ \ j = 0, 1, .., N_s - 1$$

$$\bar{x} = (x^+ + x^-)/2$$

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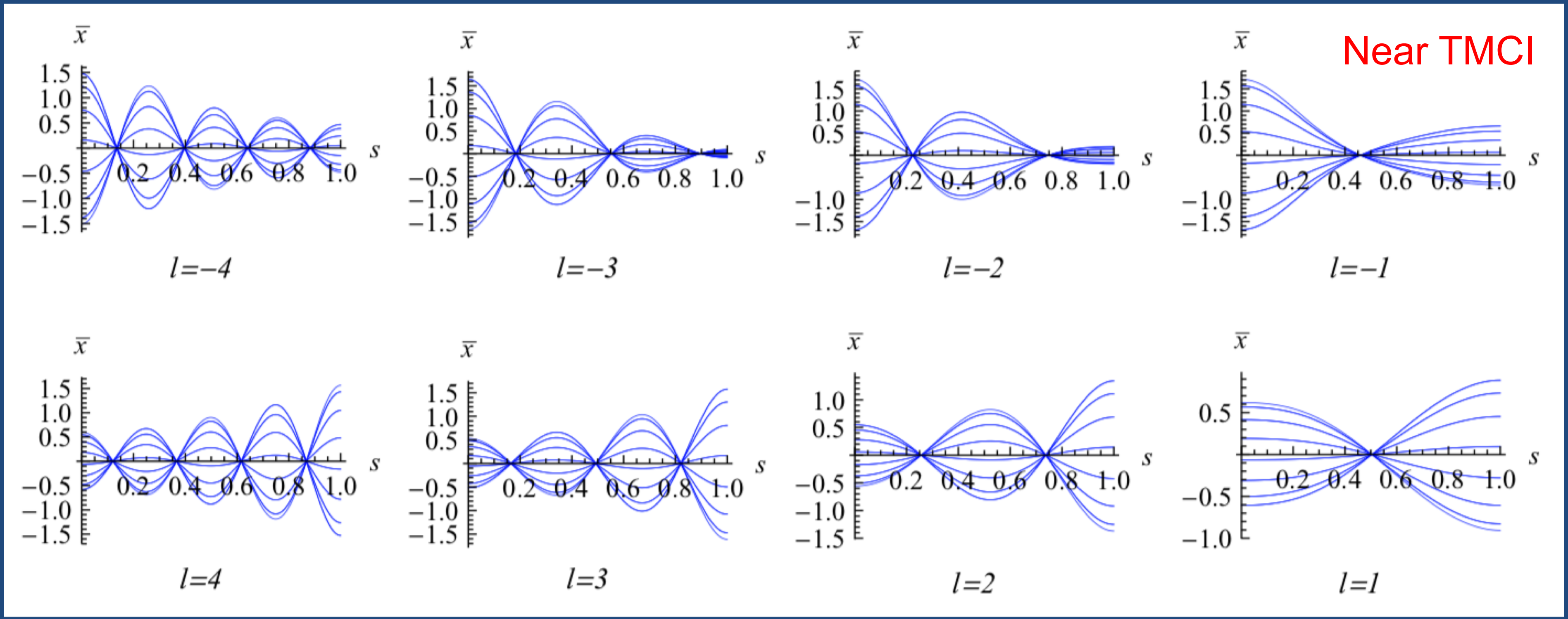
Eigen-Centroids, No SC

$$W(s) = \exp(-\alpha s)\sin(\bar{k}s) \qquad \alpha = k_r/(2Q_r) \qquad \bar{k} = \sqrt{k_r^2 - \alpha^2}$$



Modes -2 and -3 are about to couple

$k_r = 10 \qquad Q_r = 1 \qquad w = 13$

At no-SC, $w_{th} = 15$

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Centroid oscillations, strong-strong case

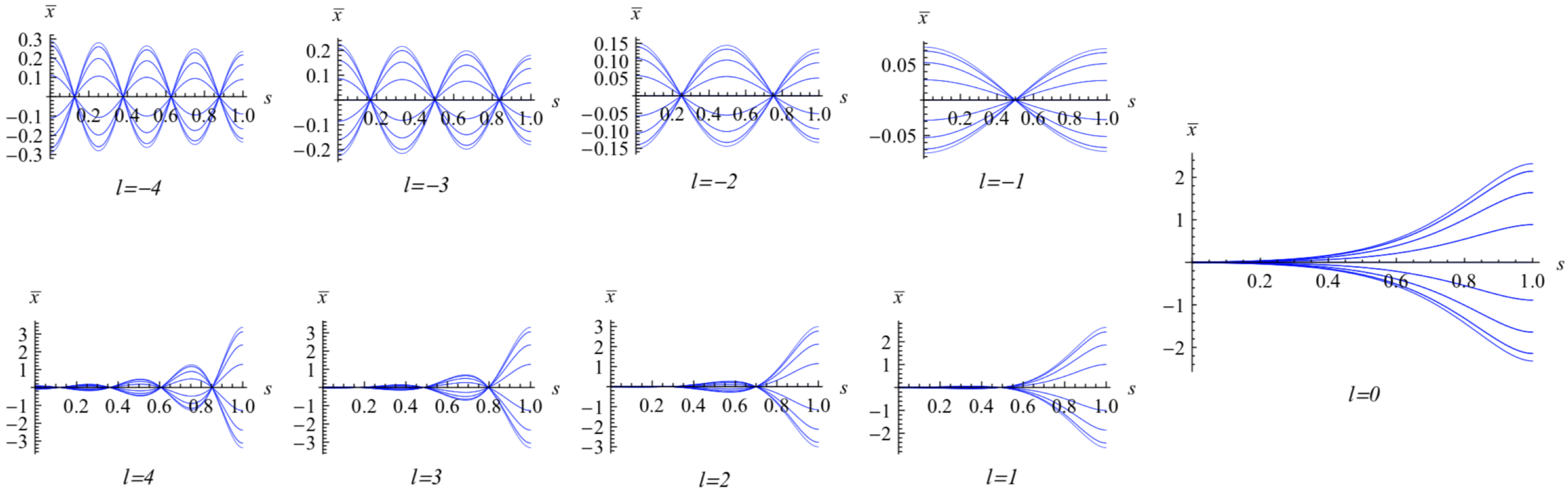

FIG. 6. Stroboscopic images of the centroid oscillations for the same parameters and modes as in Fig. 5. Number of nodes for each mode is identical to the modulus of its number.

$$q = 20 \qquad w = 13$$

Threshold w=115 at this q

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Same case

Wake & Strong SC

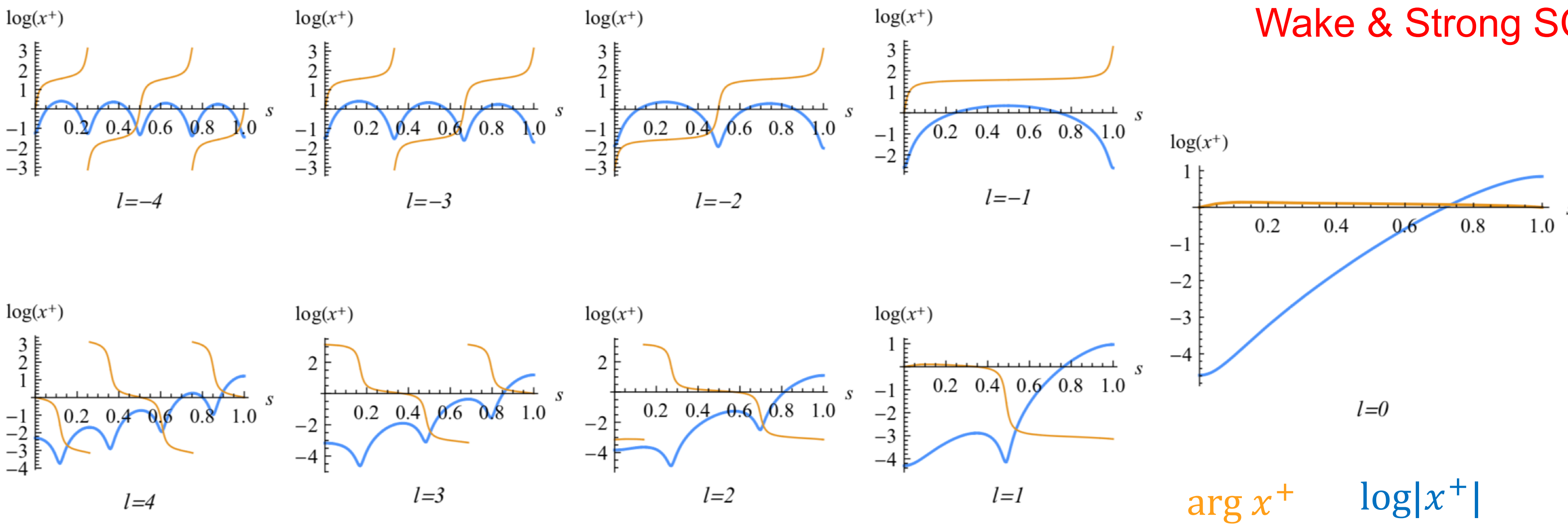

FIG. 5. Eigenfunctions with the broadband resonator wake, Eq. (10), wake and SC parameters $w = 13$, $q = 20$; compare with Figs. 1, 3. At that strong SC, the wake parameter w is $\simeq 9$ times below the TMCI threshold. Blue lines show natural logarithms of the amplitudes $\log|x_l^+|$; the orange ones are reserved for the phases $\arg(x_l^+)$. All the modes are absolutely stable, $\Im\nu_l = 0$, while head-to-tail amplification for the non-negative modes may exceed 100 for these parameters; note the *cobra shapes*, typical for these convective instabilities. Contrary to that, the negative modes look identical to their no-wake shapes of Fig. 1: with the out of phase motion of the + and − fluxes, the wake fields of the fluxes almost cancel each other.

$q = 20$ $\qquad w = 13$

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Centroid, above the TMCI

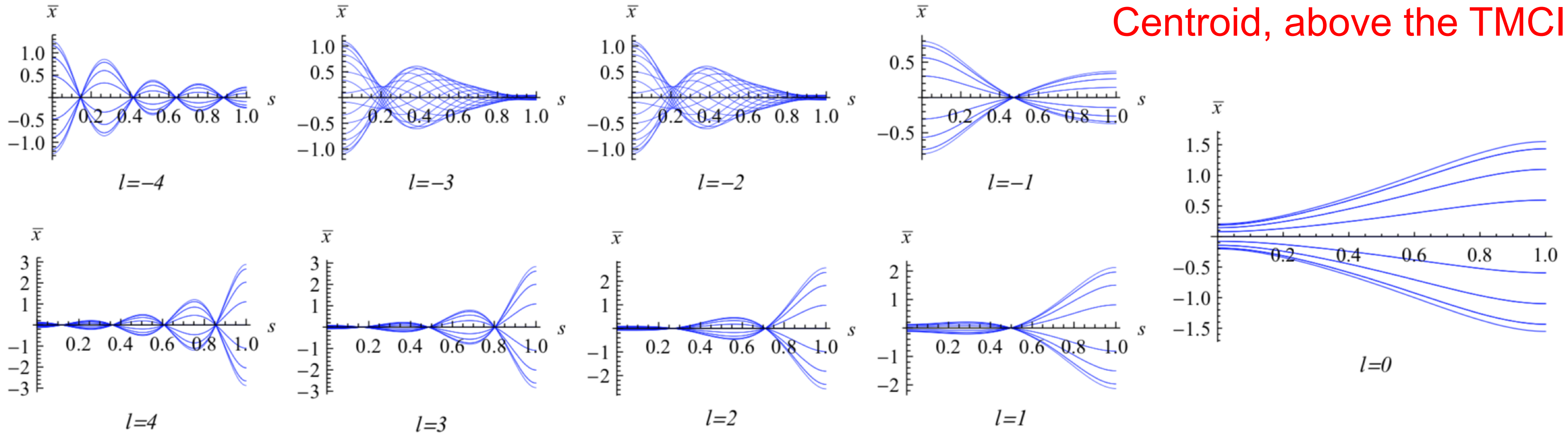
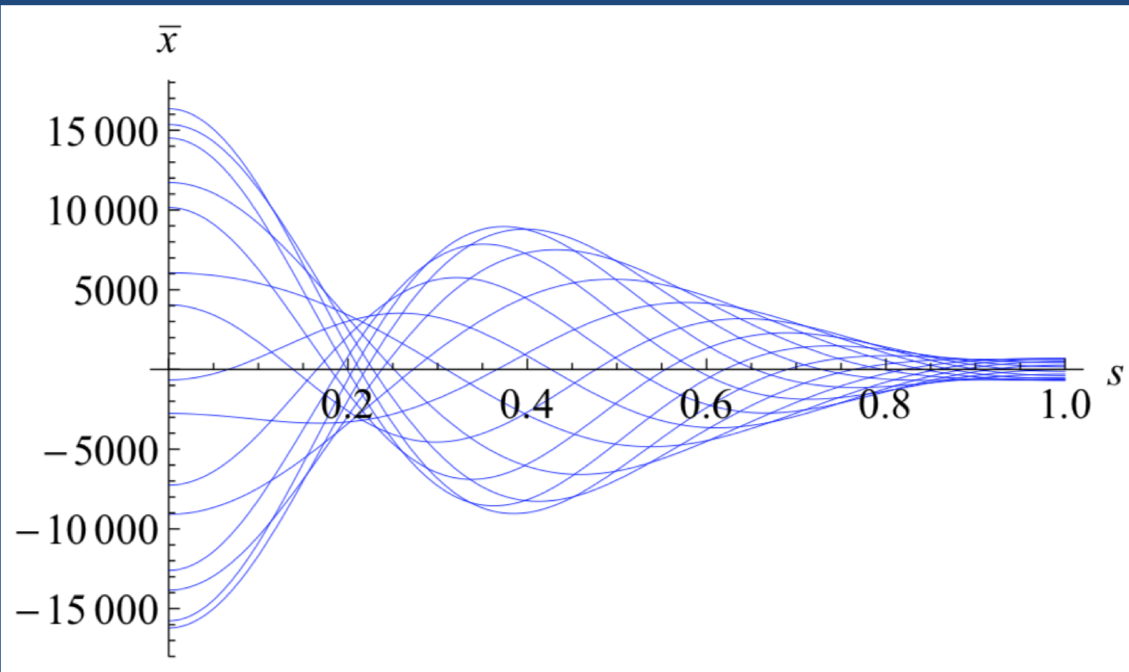


FIG. 7. Centroid oscillations for a moderate SC, $q = 4.1$ and $w = 35$, which is a bit above the TMCI threshold $w_{\rm th} = 30$ at this SC parameter, twice as high as at zero SC. Nodes of the coupled modes $l = -2$ and $l = -3$ become waists. Note that head-dominated TMCI of the negative modes is complemented by tail-dominated SCI of the positive ones.



$q = 4$ $w = 35$

FIG. 10. Evolution of the standard initial conditions $x^{\pm} = 1$ after 8 synchrotron periods for the same wake and SC parameters as in Fig. 7, $q = 4.1$ and $w = 35$, slightly above the TMCI threshold wake value $w_{\rm th} = 30$ at this SC, twice as it is at no SC case. Identity of this pattern with the coupled eigenfunctions $l = -2$ and $l = -3$ of Fig. 7 serves as a good cross-check.

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Cauchy Problem

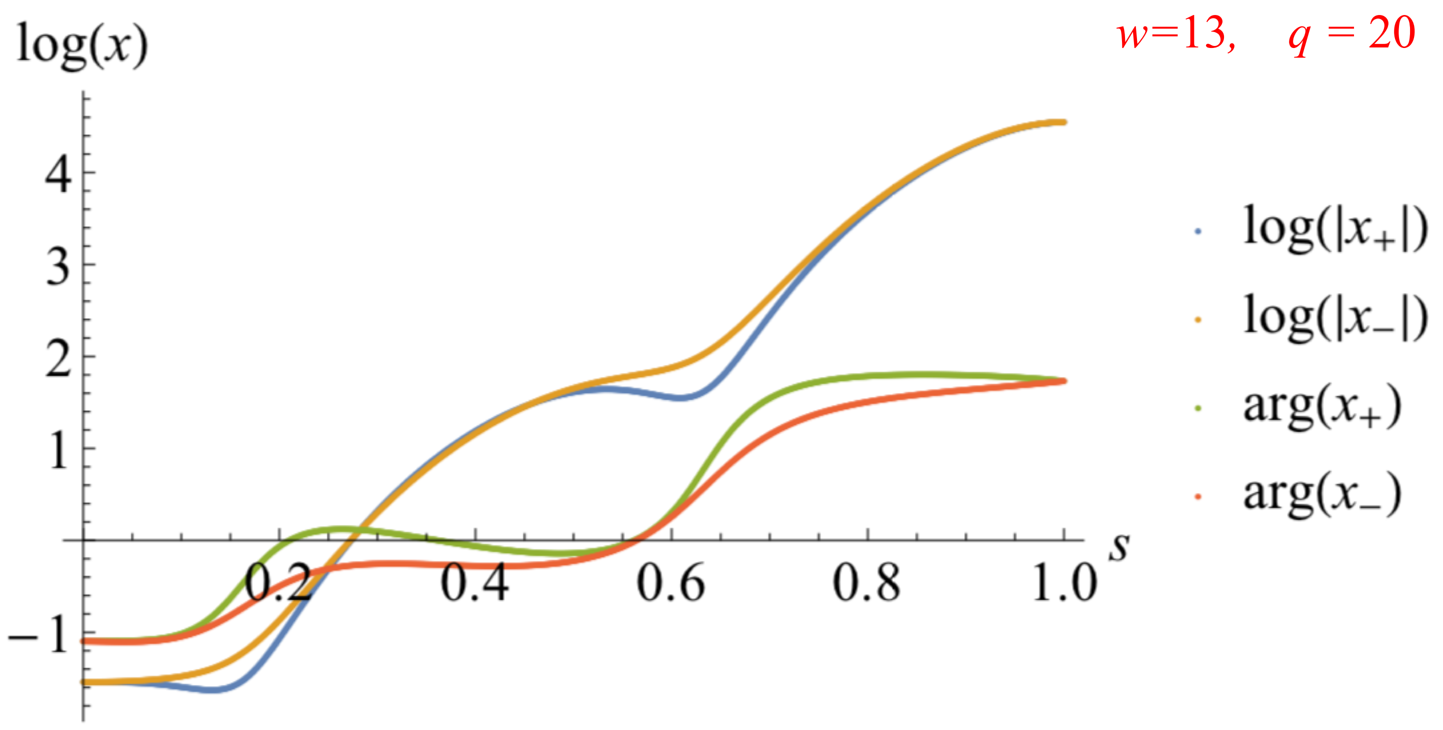

FIG. 11. Evolution of the standard initial perturbation after 1.5 synchrotron periods for the wake parameter $w = 13$ and strong SC, $q = 20$. Note that the two amplitudes are close; compare with Fig. 5.

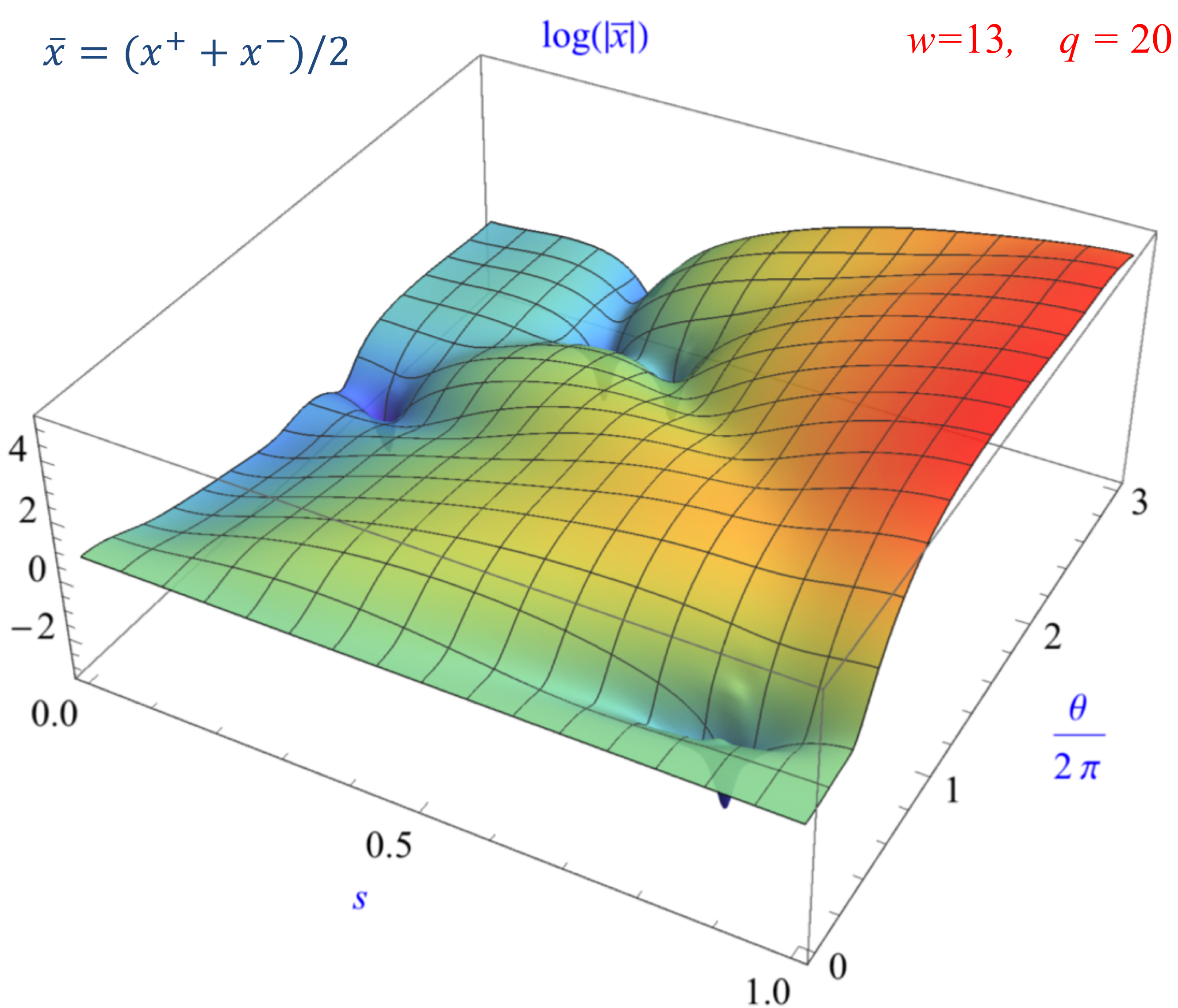

FIG. 12. Time evolution of the local centroids $\bar{x}(\theta, s) = [x^+(\theta, s) + x^-(\theta, s)]/2$ for the same case, i.e. for $q = 20$, $w = 13$ and constant initial conditions, $x^\pm = 1$. The amplification is saturated within ~1 synchrotron period.

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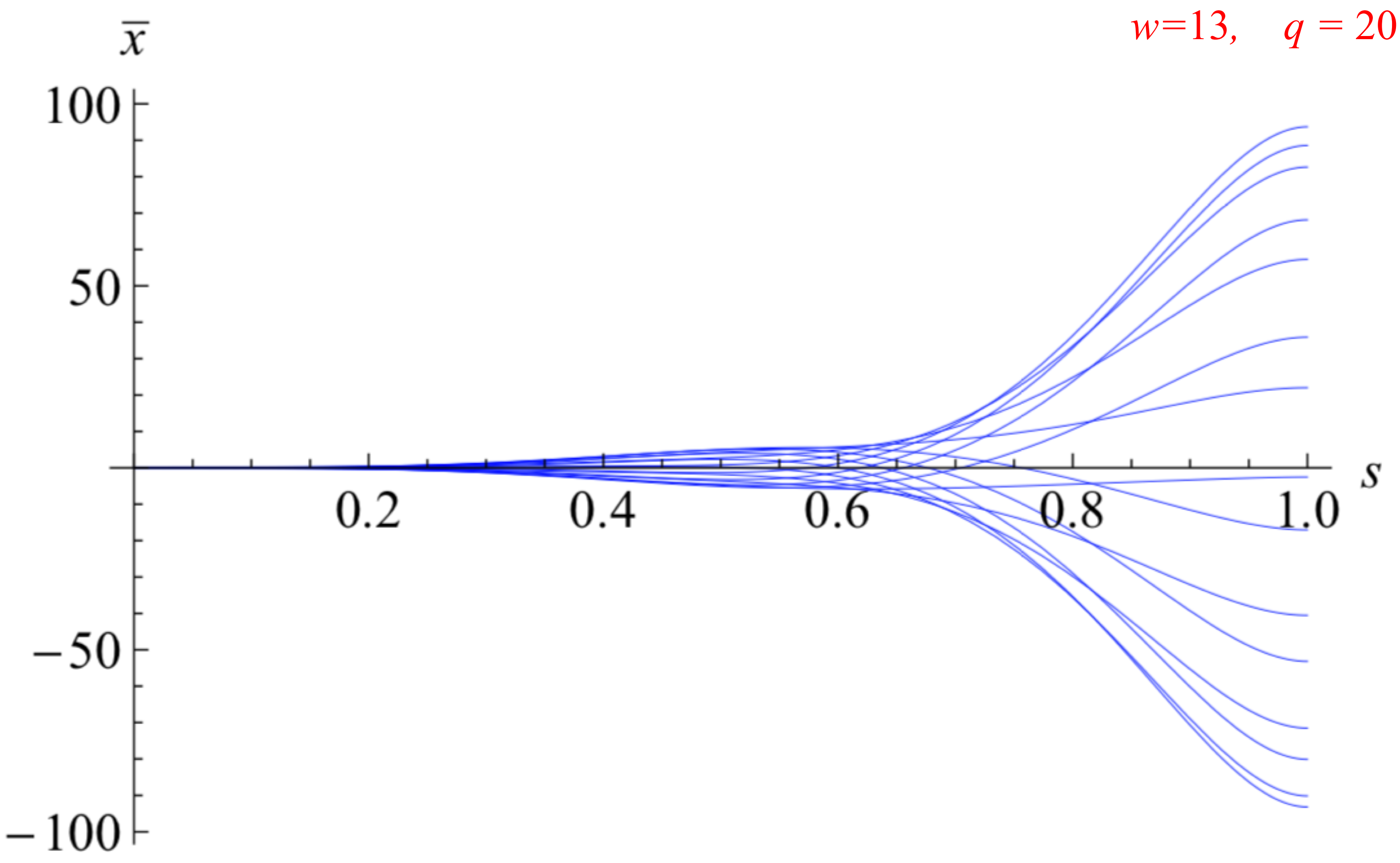

FIG. 13. Stroboscopic image of the beam centroid for the same parameters as in Fig. 12 after 1.5 synchrotron periods. Note that there are no nodes.

Absolute-Convective Instability (ACI)

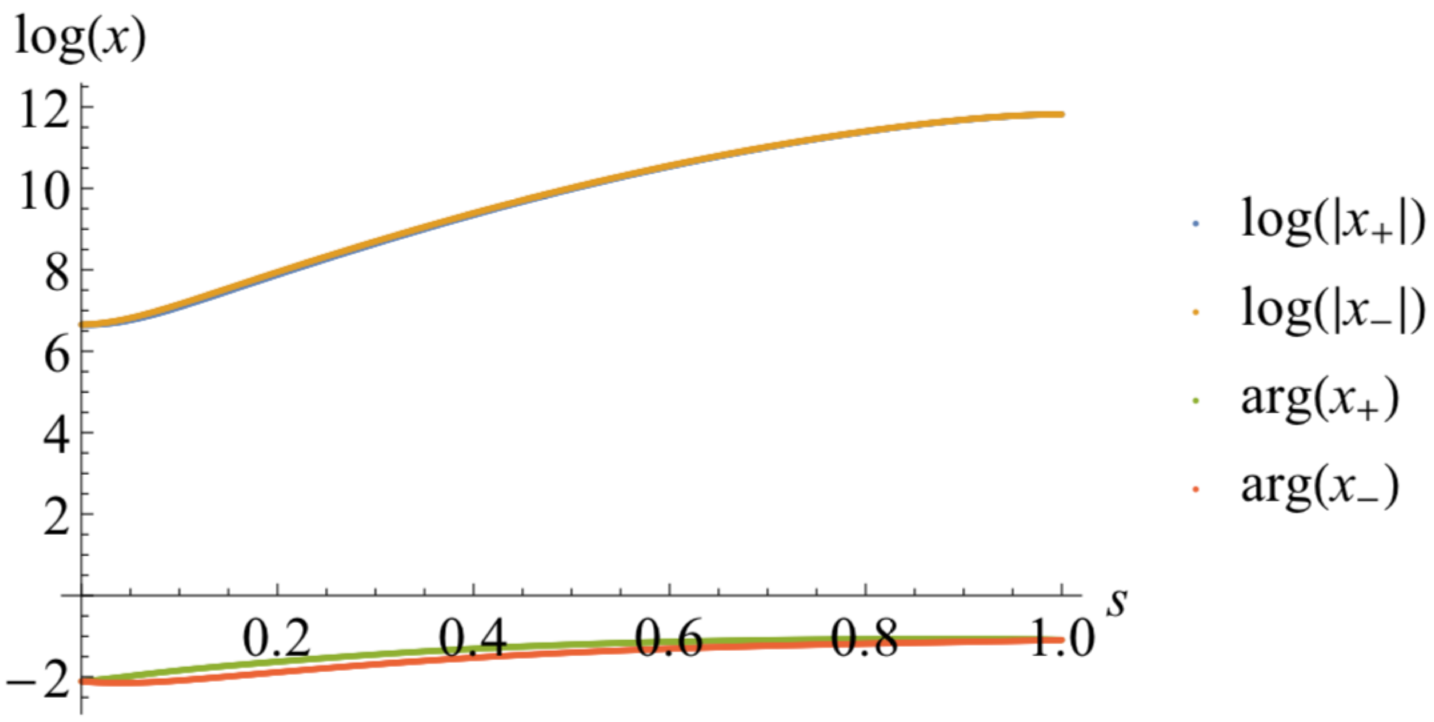

FIG. 15. Evolution of the constant initial conditions $x^{\pm}(s) = 1$ after time $\theta = 32 \cdot 2\pi$, or 32 synchrotron periods, with the gain so small that $g\,\theta = 1$. The growth rate is $\sim$6 times higher than what the gain provides by itself. The wake phase advance $k_r = 10$, the SC parameter $q = 20$, the wake parameter corresponds to the no-SC TMCI threshold, $w = 15$.

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ACI excited by damping

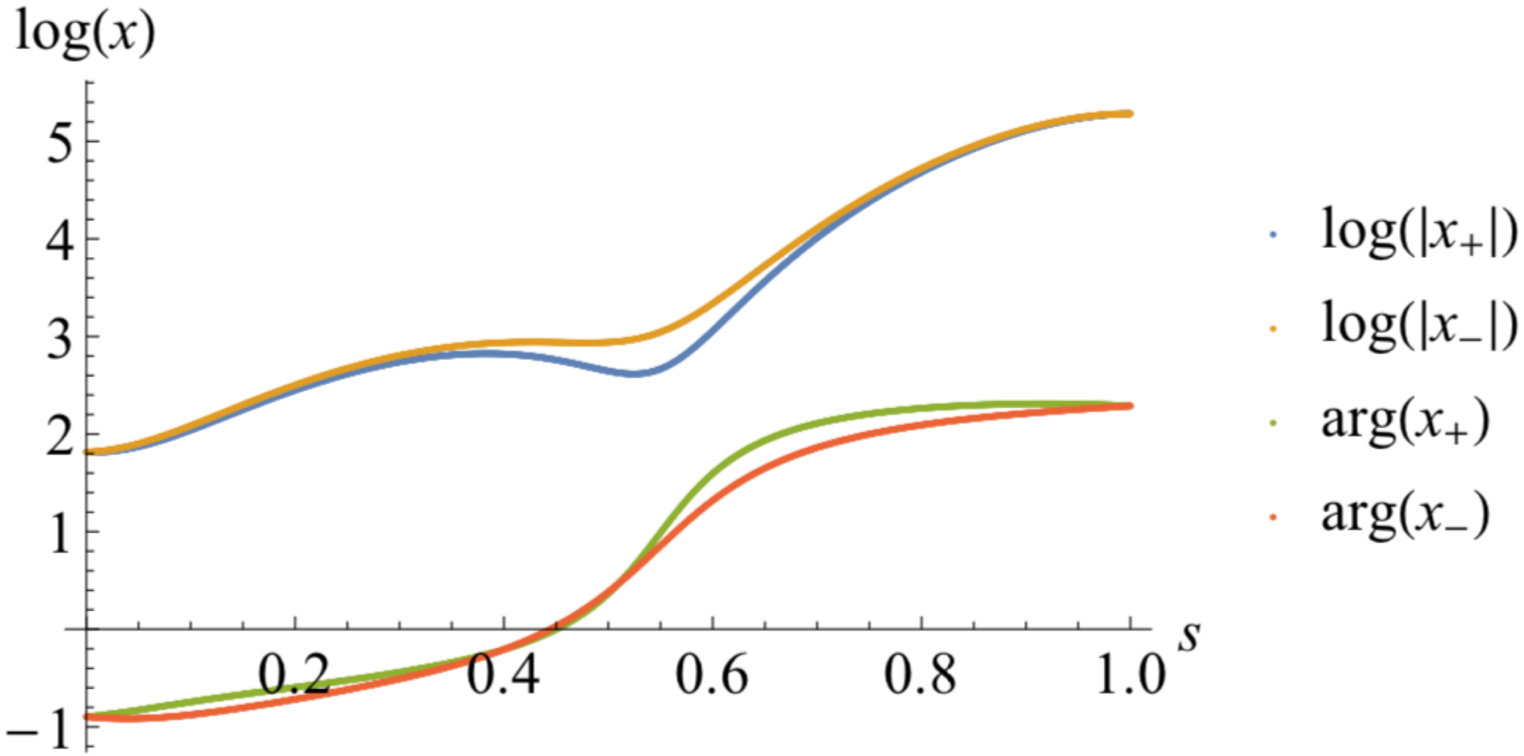

FIG. 16. ACI driven by damping (*sic!*), with the gain $g = -0.024 = -0.15/T_s$, for the wake parameter $w = 7$ and SC parameter $q = 20$. Evolution of the initial constant offset $x^{\pm} = 1$ is shown after 10 synchrotron periods. Pure convective instability, SCI, for these wake and SC parameters is shown in Fig. 14.

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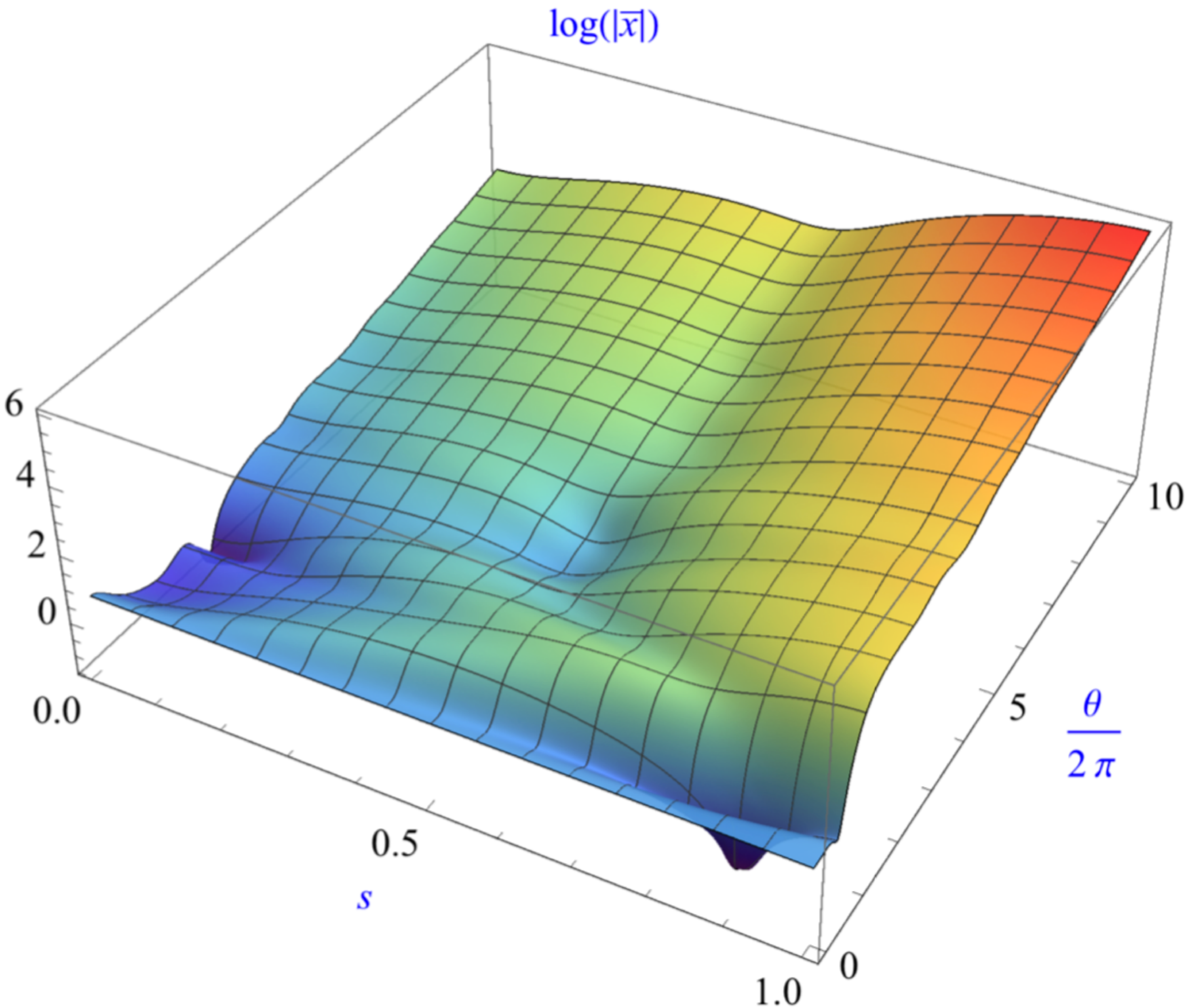


FIG. 17. Time evolution of the ACI for the same parameters as Fig. 16. An exponential growth is clearly seen.

PS Observations

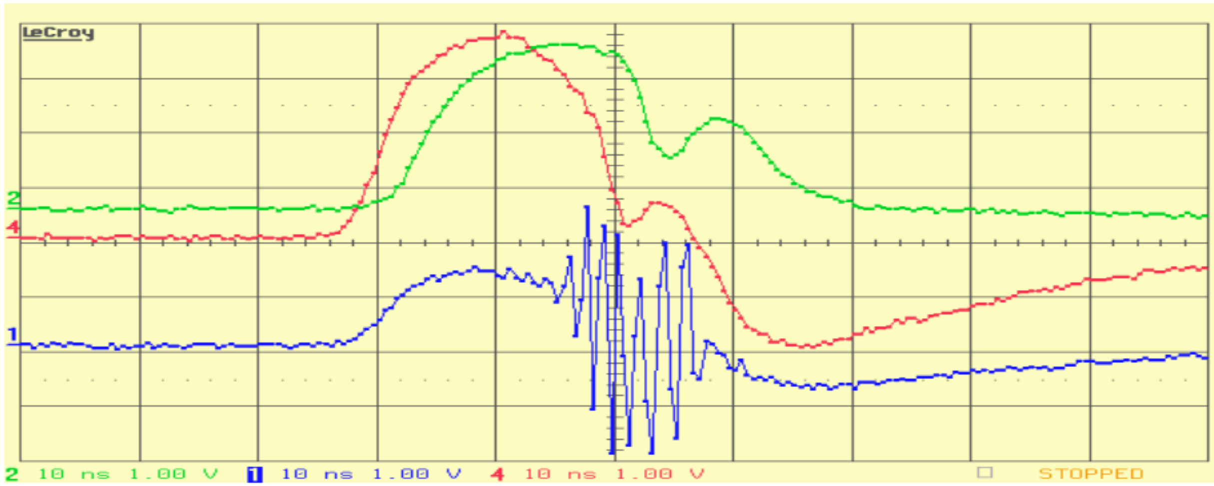
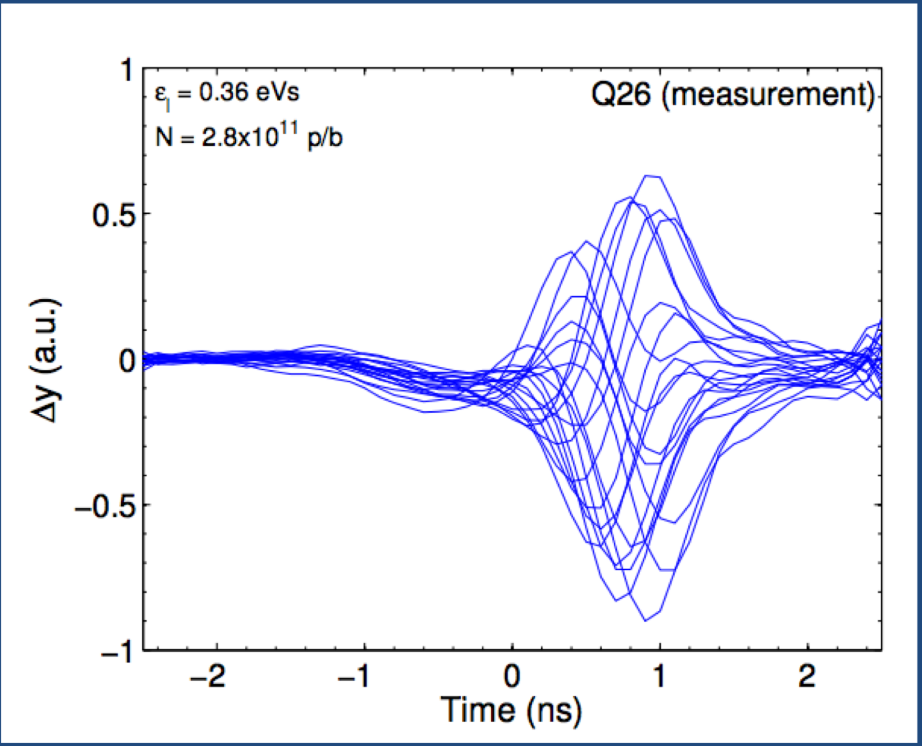
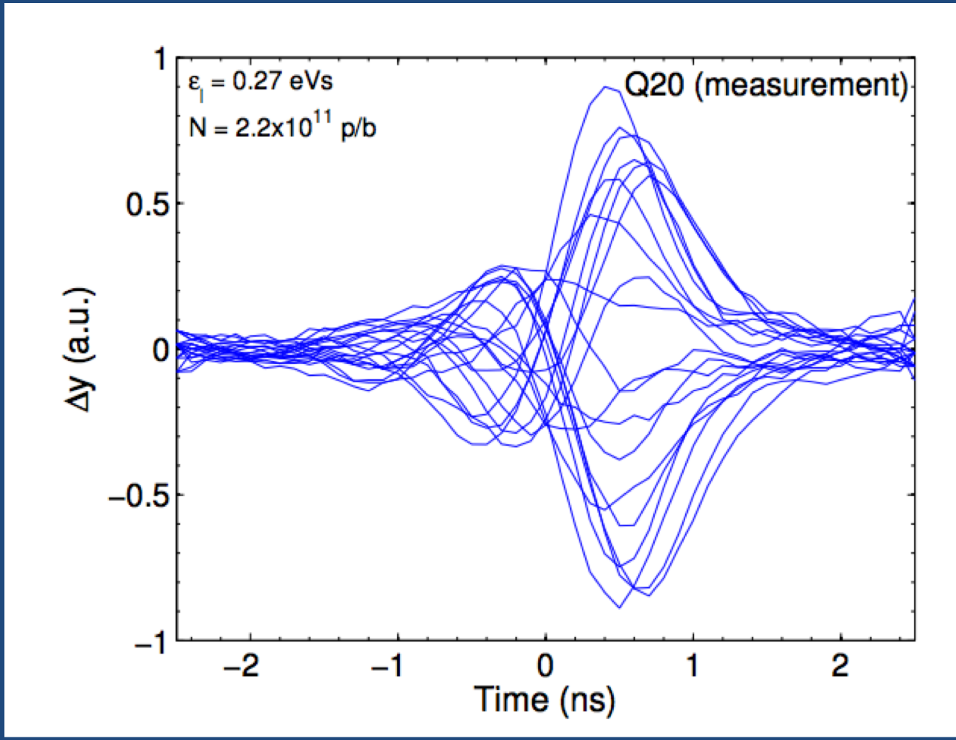


Figure 4: Fast instability observed in the CERN PS near transition (~6 GeV total energy) in 2000. Single-turn signals from a wide-band pick-up. From top to bottom: Σ, Δx, and Δy. Time scale: 10 ns/div. The head of the bunch is stable and only the tail is unstable in the vertical plane. The particles lost at the tail of the bunch can be seen from the hollow in the bunch profile.

E. Metral, HB 2005

SPS Observations





H. Bartosik, CERN, Thesis, 2013

Amplification and Growth Rate, BB wake

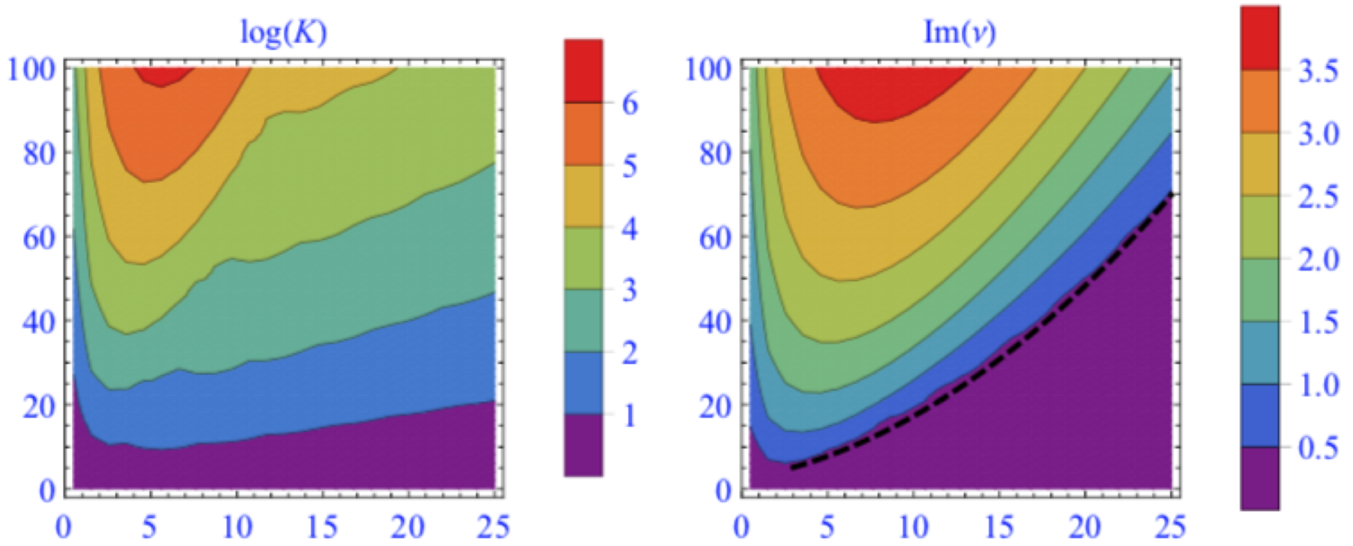


FIG. 18. Left: Contour plot for natural logarithm of the maximal head-to-tail amplification $\log K$ versus wake phase advance k_r, horizontally, and its amplitude parameter w, vertically, for the broadband case, $Q_r = 1$, and no SC. Right: TMCI growth rate for the same parameters; the black dashed line is the no-SC TMCI threshold, Eq. (18), according to Ref. [11].

No SC

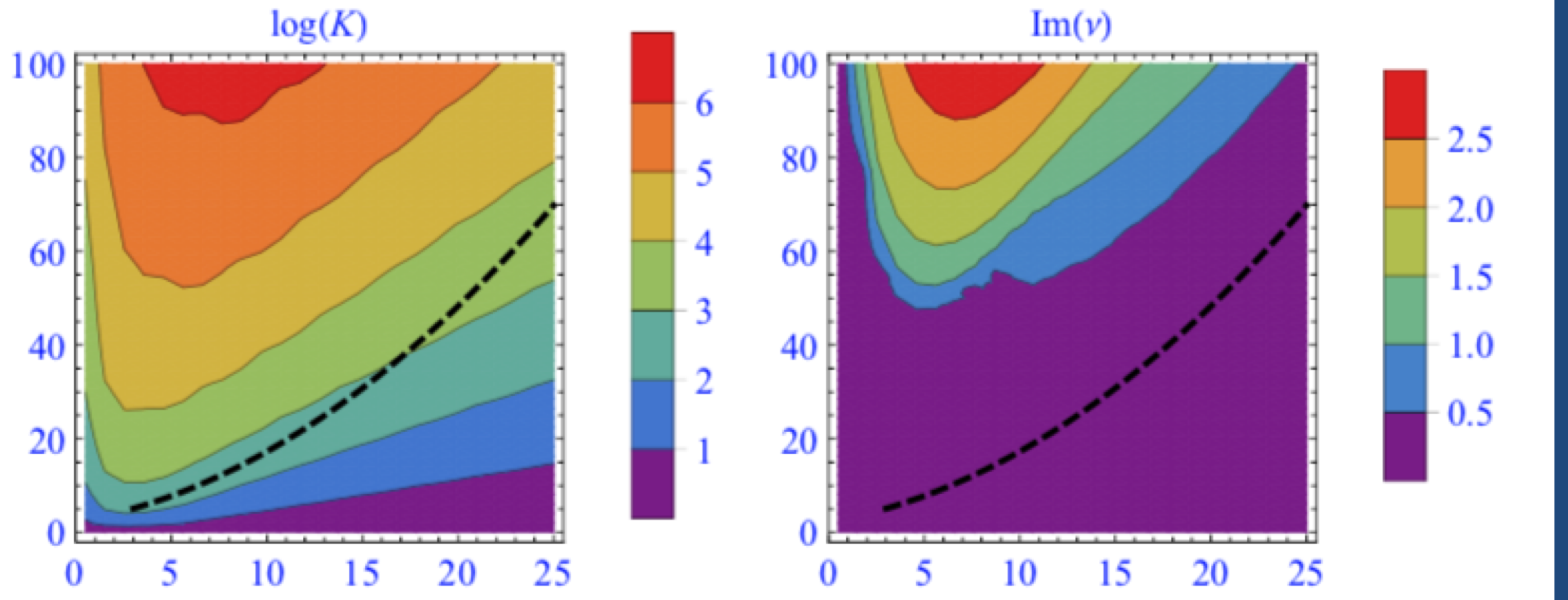


FIG. 19. Same as Fig. $\boxed{18}$, for SC $q = 5$. The TMCI threshold moves up with SC. The black dashed line is the same no-SC TMCI threshold, Eq. ($\boxed{18}$) .

$q = 5$

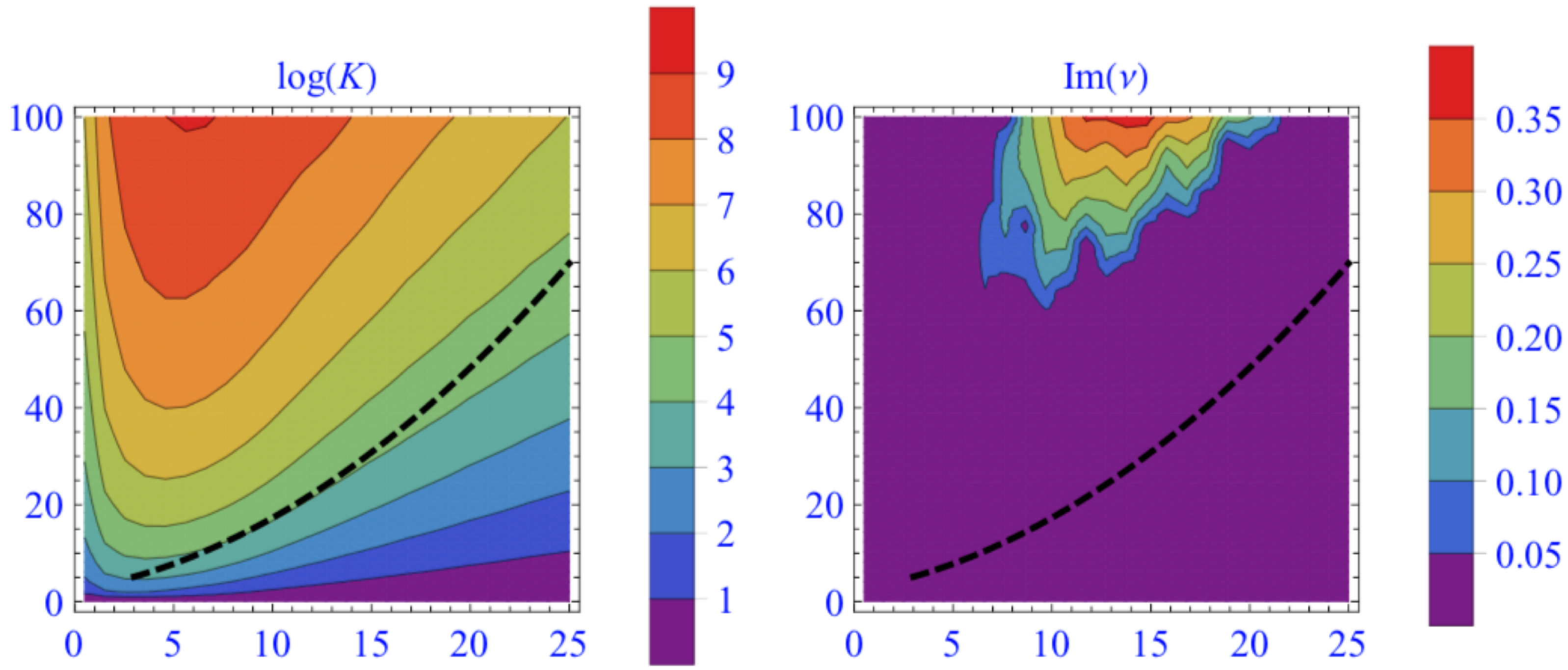


FIG. 20. The same, for larger SC, $q = 10$. While TMCI threshold recedes, the amplification grows.

$q = 10$

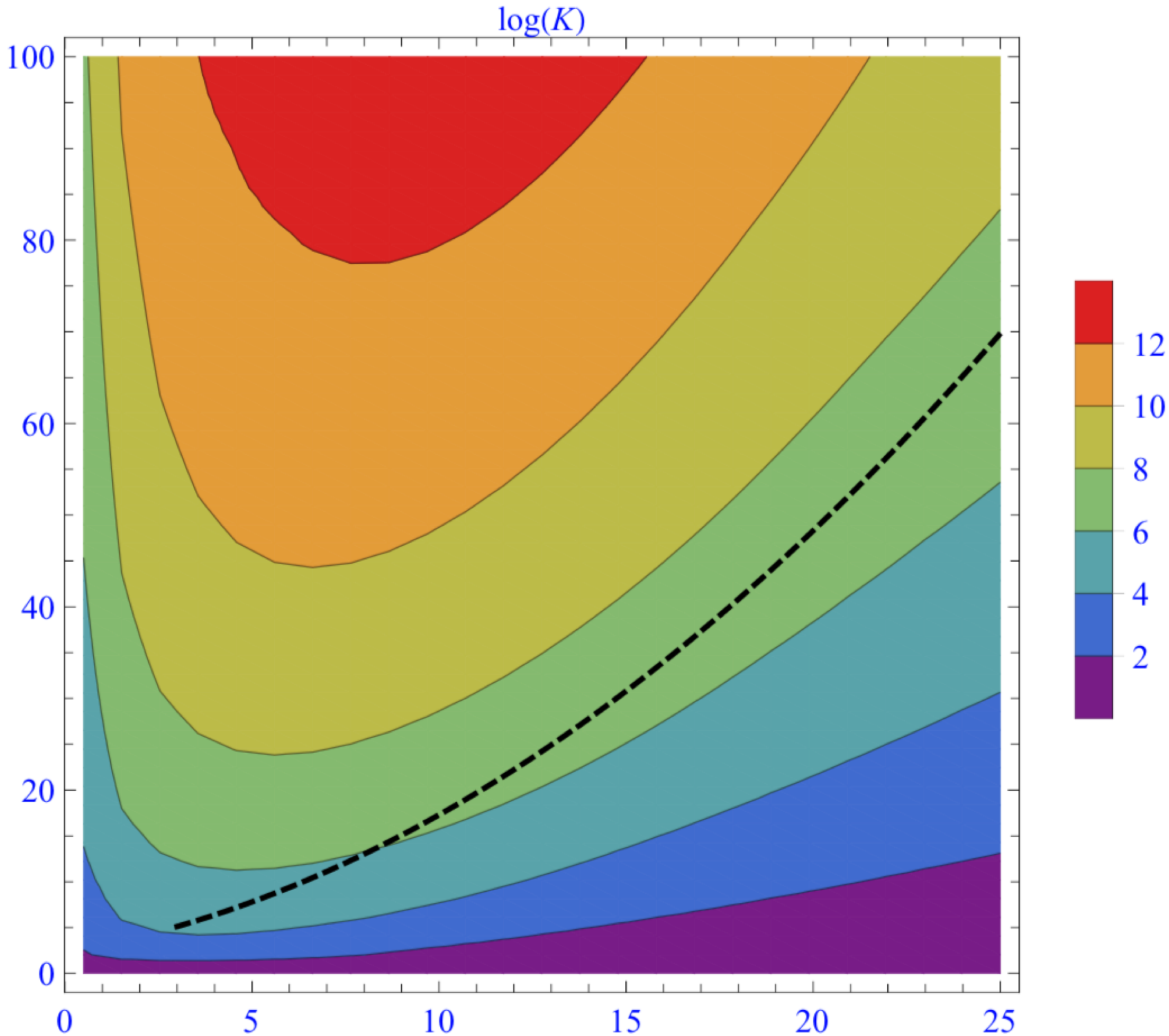


FIG. 21. Amplification for SC $q = 20$. The black dashed line of no-SC TMCI threshold is close to the contour line $K \simeq 300 - 1000$ for large interval of the phase advances. For the entire area of the parameters, the system is absolutely stable, $\Im\nu = 0$.

$q = 20$

AB

Core-Halo Collective Instabilities

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(Dated: August 28, 2018) https://arxiv.org/pdf/1808.08498.pdf

At strong space charge, transverse modes of the bunch core may effectively couple with those of the halo, leading to instabilities well below the core-only transverse mode-coupling threshold.

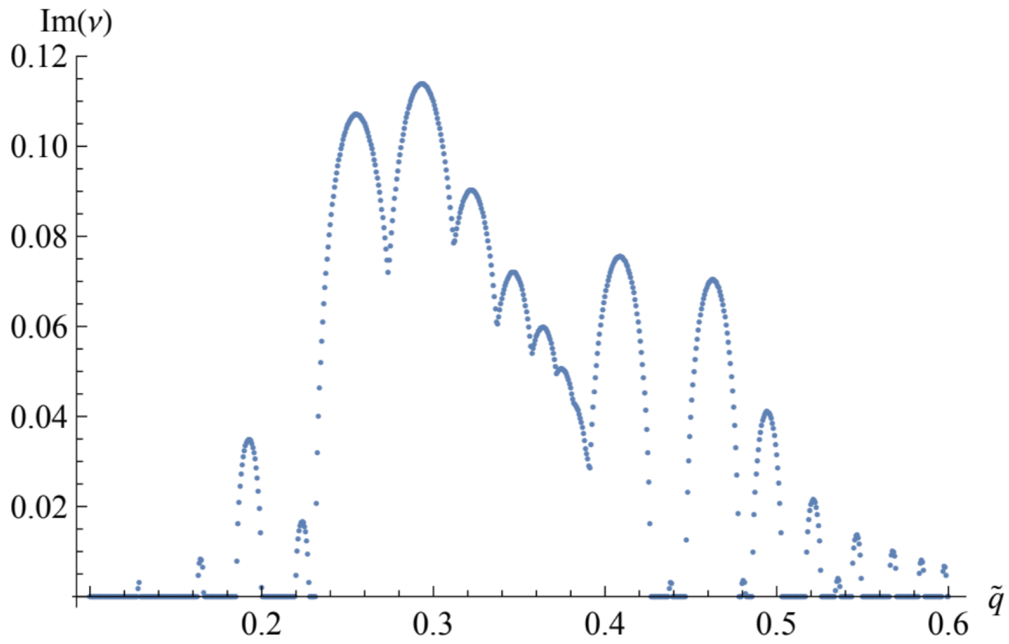

FIG. 1. Instability growth rate versus the halo parameter $\tilde{q} = q_h/q_c$ for the SC and wake $q_c = 10$ and $w = 4$.

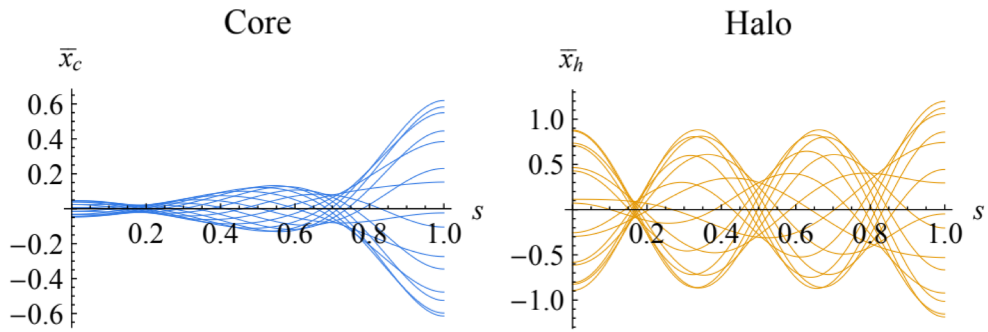

FIG. 2. Centroid stroboscopic images of the core and halo components of the most unstable core-halo mode for the same q and w as in Fig. 1, at the most unstable $\tilde{q} = 0.29$. Waists instead of nodes in the halo image tell about an absolute instability.

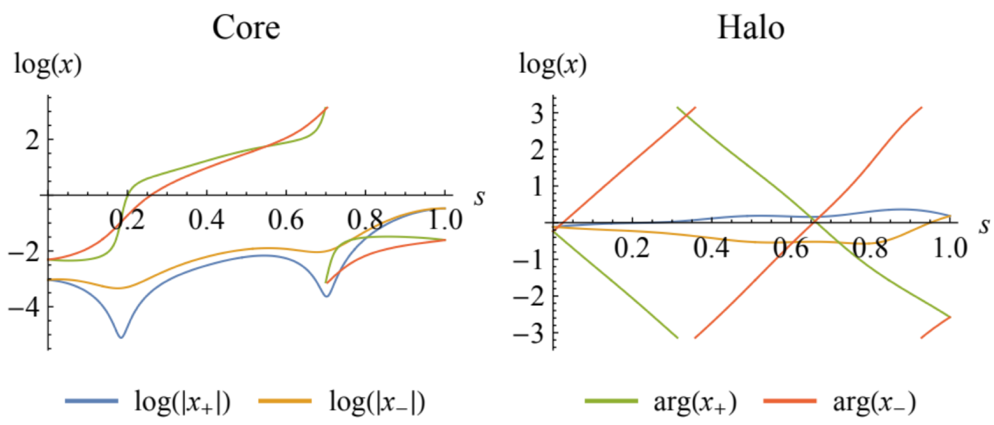

FIG. 3. Amplitudes and phases for the two fluxes of the core and the halo for the same modes, +2 and +3 correspondingly, as in Fig. 2. The core mode is convectively unstable, with its + and − fluxes in phase, while the halo mode is similar to a typical no-SC modes having the + and − phases steadily running with opposite signs.

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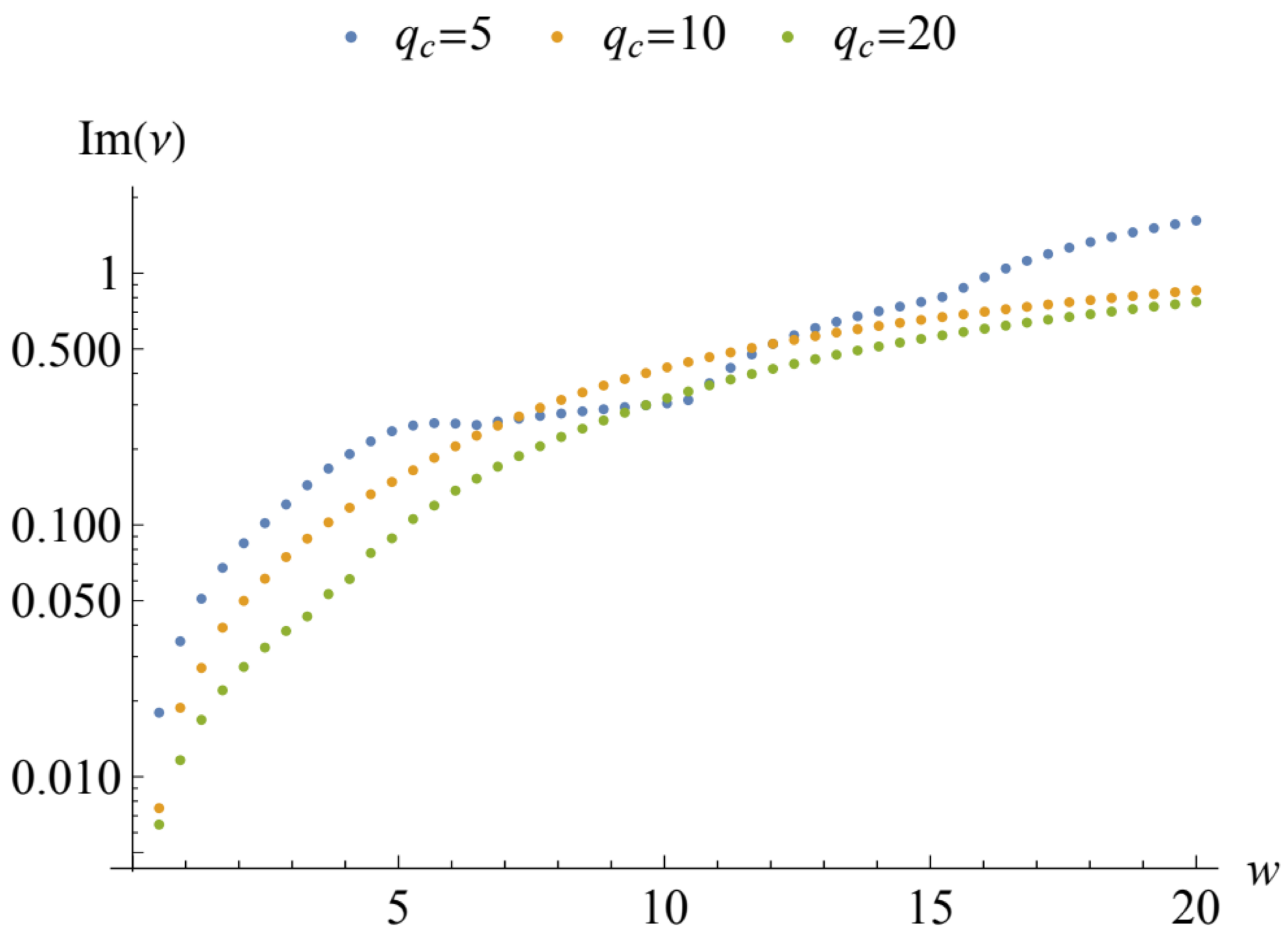

FIG. 4. Growth rates of the most unstable modes versus wake parameter for three different SC parameters. Note the conventional TMCI threshold for $q_c = 5$ at $w \approx 15$.

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Transverse Instabilities of a Bunch with Space Charge, Wake and Feedback

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(Dated: September 20, 2018)

When a resistive feedback and single-bunch wake act together, it is known that some head-tail modes may become unstable even without space charge. This **f**eedback-**w**ake **i**nstability, FWI, modified by space charge to a certain degree, is shown to have a special single-maximum increasing-dropping pattern with respect to the gain. Also, at sufficiently large Coulomb and wake **f**ields, as well as the **f**eedback gain, a new type of transverse **m**ode-**c**oupling **i**nstability is shown to take place, 3FMCI, when head-to-tail amplified positive modes couple and the growth rate saturates with the gain.

https://arxiv.org/pdf/1809.06927.pdf

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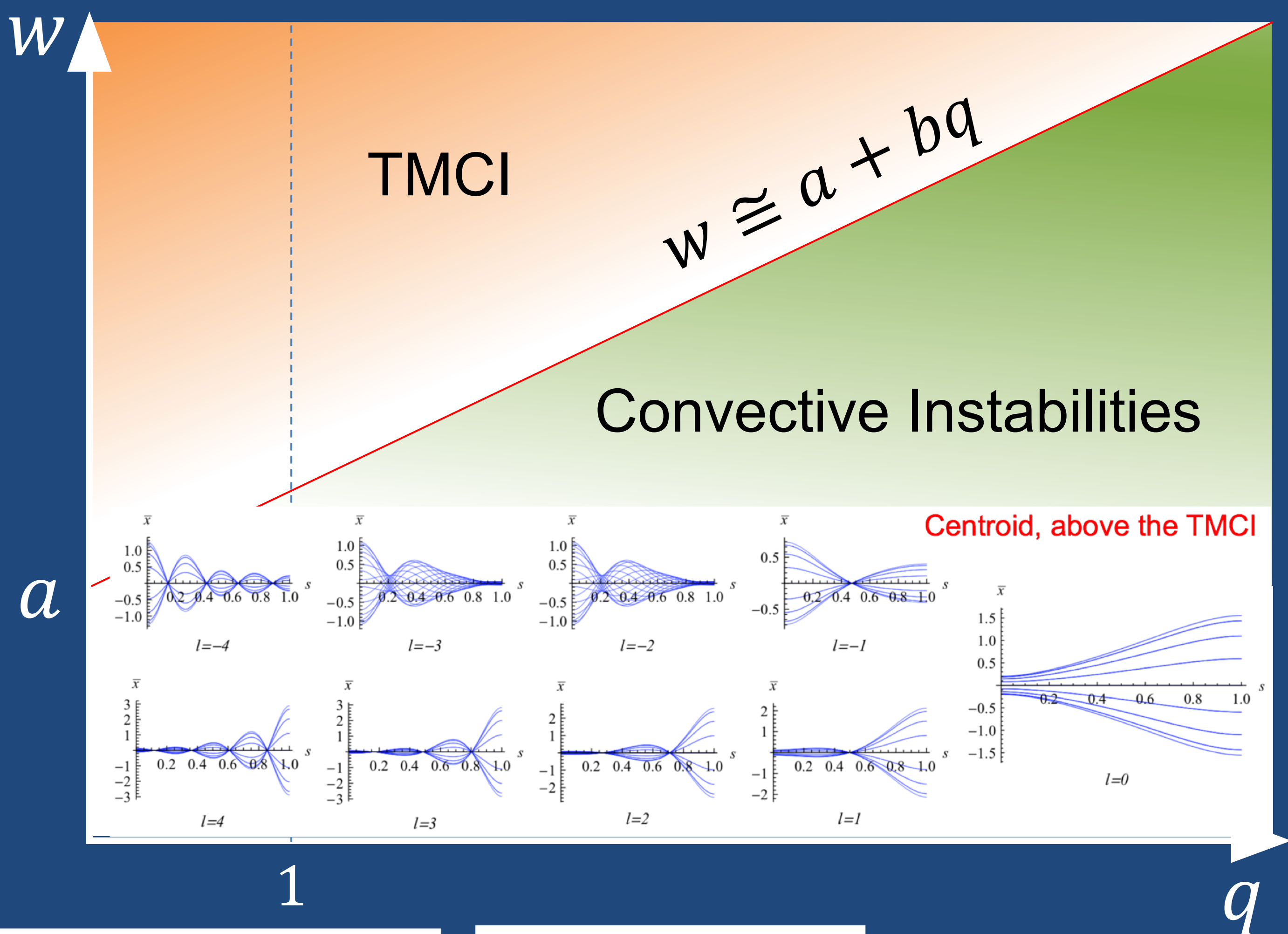
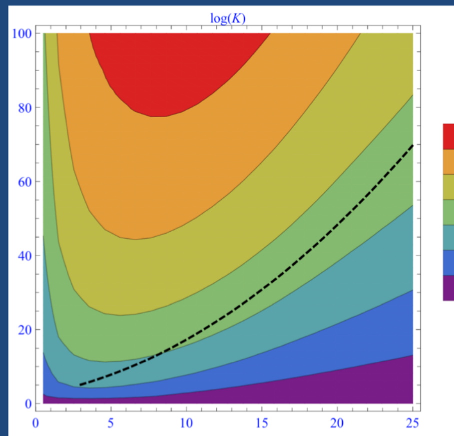
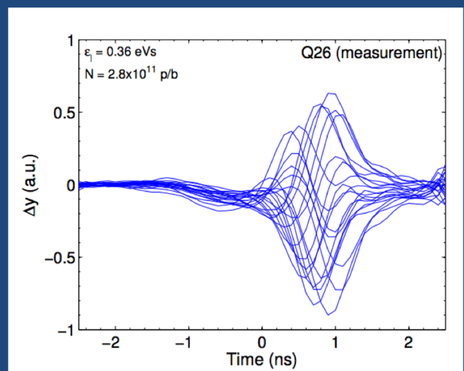
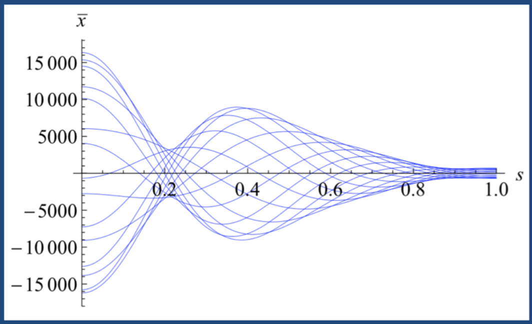
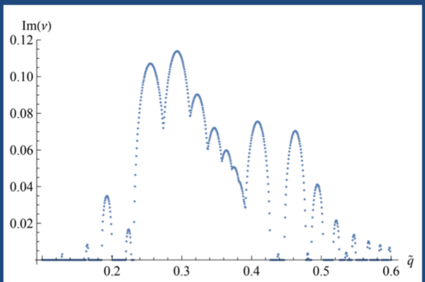
Summary











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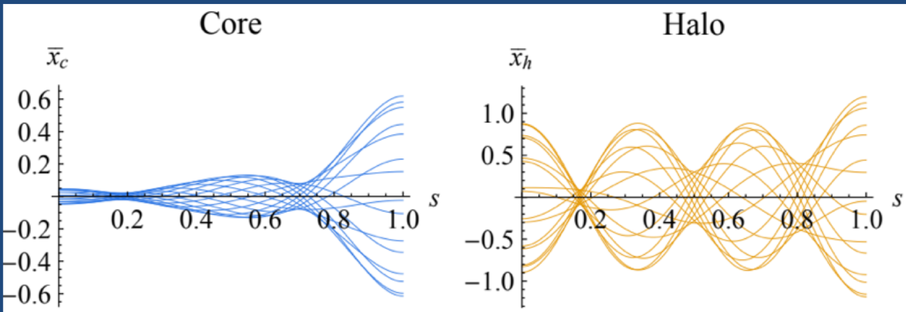

Many thanks!